The Complete Set of Noble Polyhedra

This proof used a combination of **computer-assisted** and traditional methods to find the complete list of a certain type of 3D shape known as a **noble polyhedron**, an unsolved problem in mathematics for over 100 years. It relates the existence of these figures to the **roots of polynomials** and the intersections between polynomial-related objects known as **algebraic curves**. In total, 2 infinite sets of noble polyhedra and 146 exceptional cases were found, including **85 new examples**.

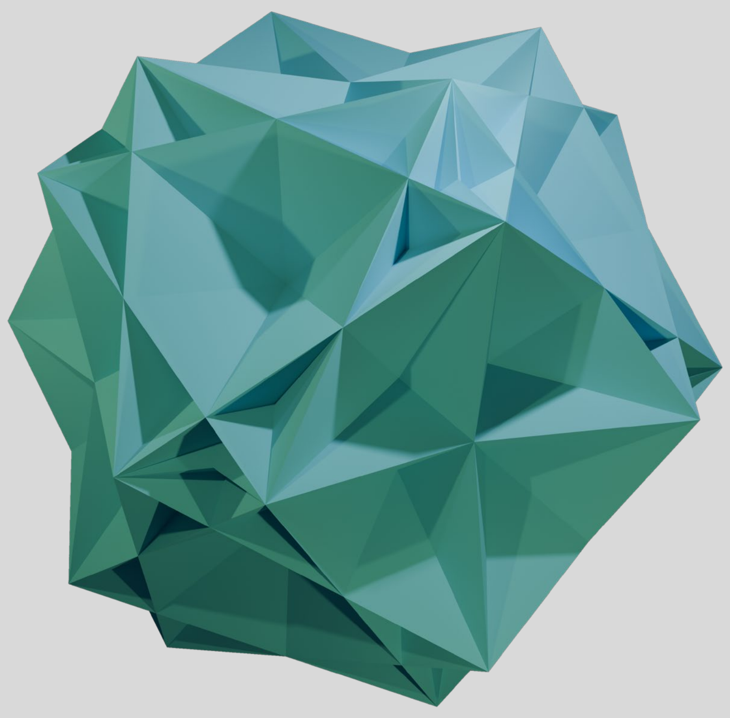

Figure 1. *tO-1.1, one of the newly discovered noble polyhedra from this proof. Graphic created by finalist using Blender, 2026.*

Introduction

Informally, a **polyhedron** is a certain type of three-dimensional shape whose sides (also called faces) are all flat, and where each edge is adjacent to two faces. We tend to like symmetric polyhedra; for example, the most famous polyhedra are the five Platonic solids, known for the fact that their corners, edges, and faces are all the same under the symmetries of each shape.

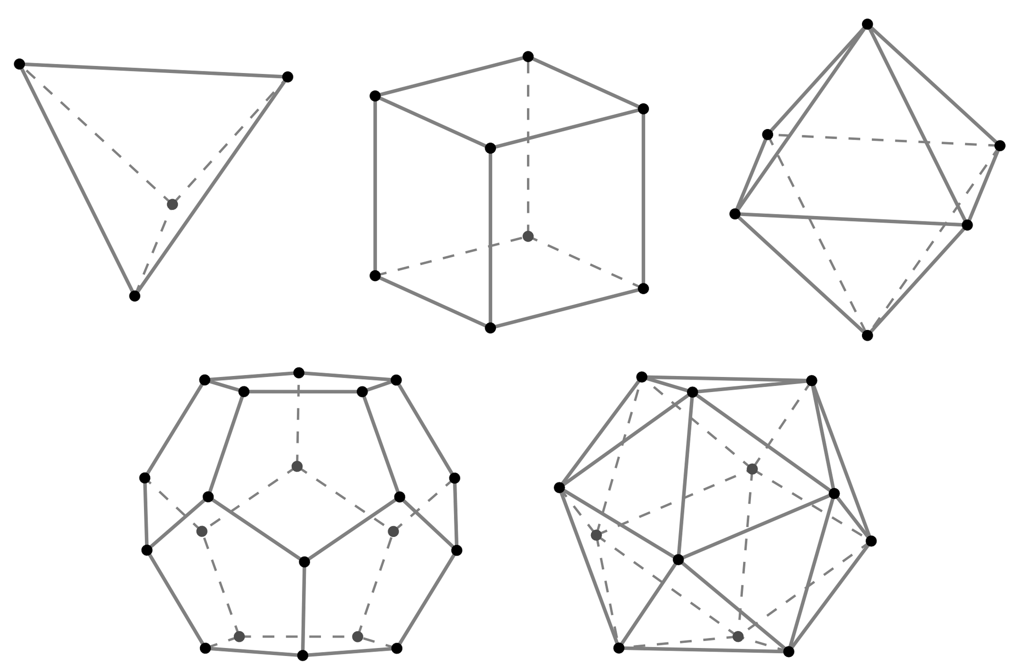

Figure 2. *The Platonic solids. Graphic created by finalist using TikZ, 2026.*

Definition

A **noble polyhedron** $\mathcal{P}$ is a polyhedron which is both vertex-transitive (all the corners of $\mathcal{P}$ are the same under its symmetries) and face-transitive (all the sides of $\mathcal{P}$ are the same under its symmetries).

If we restrict to convex polyhedra, the only noble polyhedra are the Platonic solids and a type of four-sided shape called a **disphenoid** [1]. However, when non-convex polyhedra are allowed (so parts of the polyhedron like sides and edges can intersect each other), finding all the noble polyhedra becomes a much more complex question. For my project, I completed this enumeration, finding the following main result:

Main Theorem

Other than the stephanoids (a previously known type of nonconvex noble polyhedron [2]) and disphenoids, there are exactly 146 noble polyhedra up to similarity.

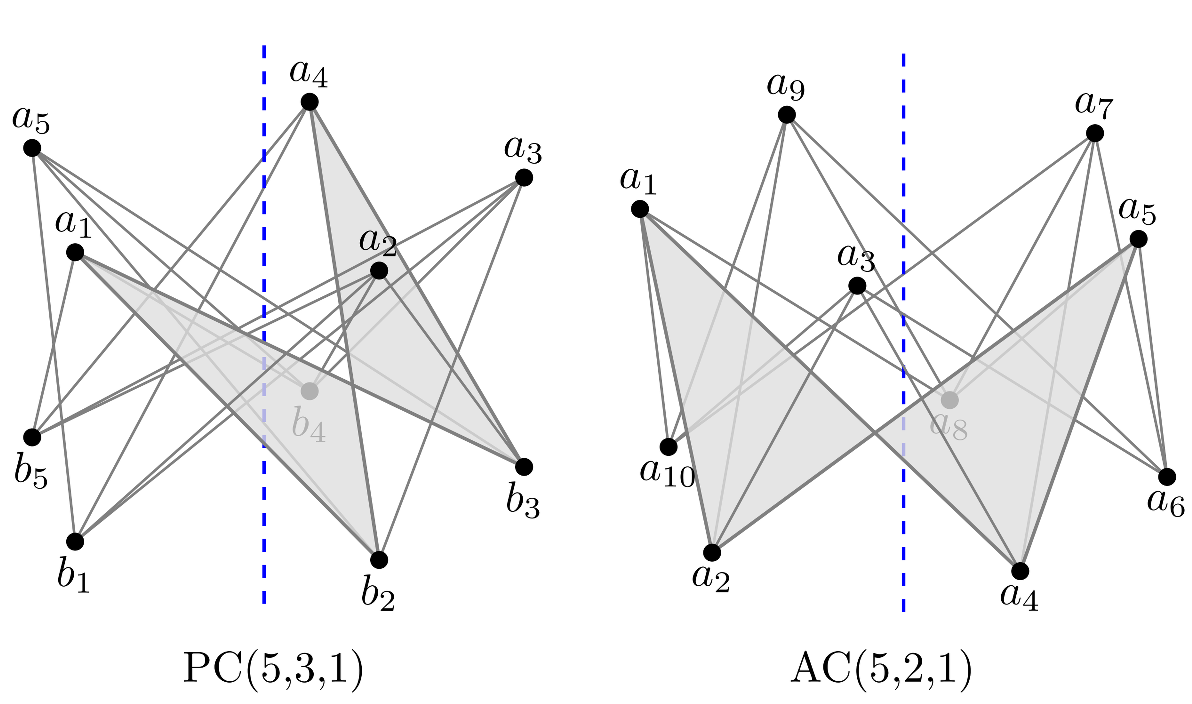

Figure 3. *Two examples of stephanoids. One face of each polyhedron is shown. Graphic created by finalist using TikZ, 2025.*

Methods

There are only so many different types of symmetry that a finite object in 3-dimensional space can have. In this proof, it is very helpful to consider the possibilities separately based on their symmetry. In particular, it matters whether the symmetries of a noble polyhedron are **prismatic** (the same symmetries as a prism) or **nonprismatic** (one of the 7 other symmetry groups which are not prismatic). The prismatic case was dealt with using traditional methods, but the nonprismatic case required much more intensive calculation. The main idea of the proof is to divide candidates for the corners of noble polyhedra into collections called **orbit types**, which then allows for the conversion of this problem into one of algebraic geometry and numerical analysis.

In order to accomplish this, I wrote a collection of computer programs which converted this geometry problem into an equivalent formulation made of smaller algebra problems involving algebraic curves. Using a computer like this to help with a mathematical proof is known as a **computer-assisted proof**.

Results

In total, the search found that there are 146 nonprismatic noble polyhedra, and in addition to the proof that the only prismatic noble polyhedra are the previously known disphenoids and stephanoids, this completes the enumeration and proof. In addition to the previously known examples, over 85 new examples were found compared to the last published list [3].

Additional Result

Interestingly, the way that this proof works also happens to show that all nonprismatic noble polyhedra can be given algebraic coordinates: i.e., for any vertex (x, y, z) of a noble polyhedron, x, y, and z are all algebraic numbers (roots of a polynomial whose coefficients are integers). I was also able to find exact coordinates for each nonprismatic noble polyhedron using this method.

References

1. Grünbaum, B. (2003). Are your polyhedra the same as my polyhedra? In B. Aronov, S. Basu, J. Pach, & M. Sharir(Eds.), Discrete and computational geometry: The goodman-pollack festschrift (pp. 461–488). Springer.
2. Grünbaum, B. (1993). Polyhedra with hollow faces. In T. Bisztriczky, P. McMullen, R. Schneider, & A. I. Weiss(Eds.), Polytopes: Abstract, convex and computational (pp. 43–70, Vol. 440). Kluwer Academic Publishers.
3. Mikloweit, U. (2020). Exploring noble polyhedra with the program Stella4D. In C. Yackel, E. Torrence, K. Fenyvesi,R. Bosch, & C. Kaplan (Eds.), Bridges 2020 conference proceedings (pp. 257–264, Vol. 25).

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