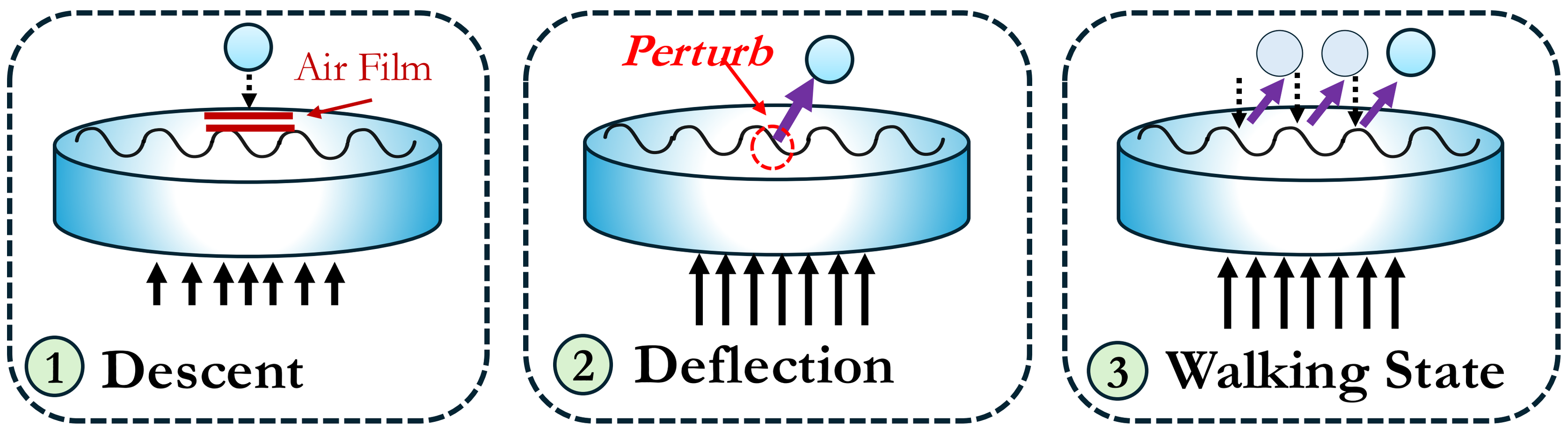
FROM WALKING TO TUNNELING: AN INVESTIGATION OF GENERALIZED PILOT-WAVE DYNAMICS

Background



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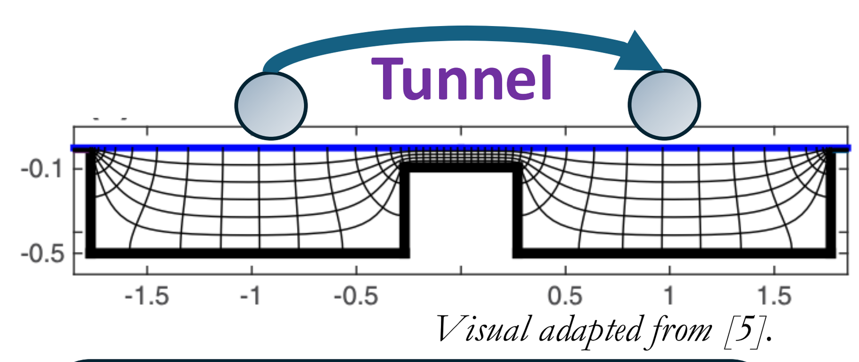
When placed above a vibrating fluid bath, **droplets of silicone oil** defy intuition by perpetually **"walking"** on the surface of the bath while generating an associated **wave-field** [1–4].

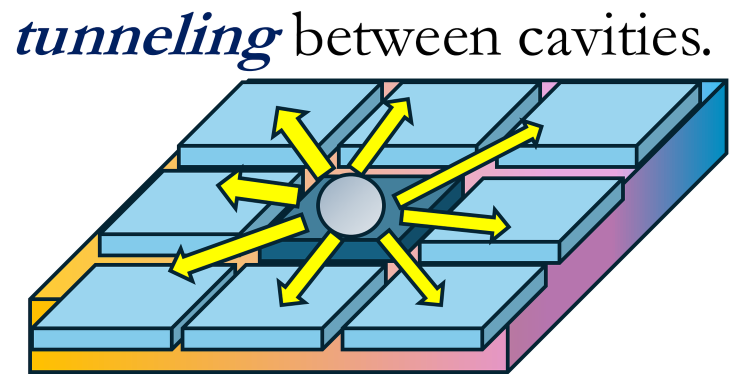
Walking droplets exhibit **robust dynamical behaviors**, many of which are **macroscopic analogues** to quantum mechanics.

Research Objectives

Motivation: A significant gap in existing literature concerns the difficulty of analyzing a **complete, three-dimensional droplet system** to study ***droplet tunneling*** between cavities.



Visual adapted from [5].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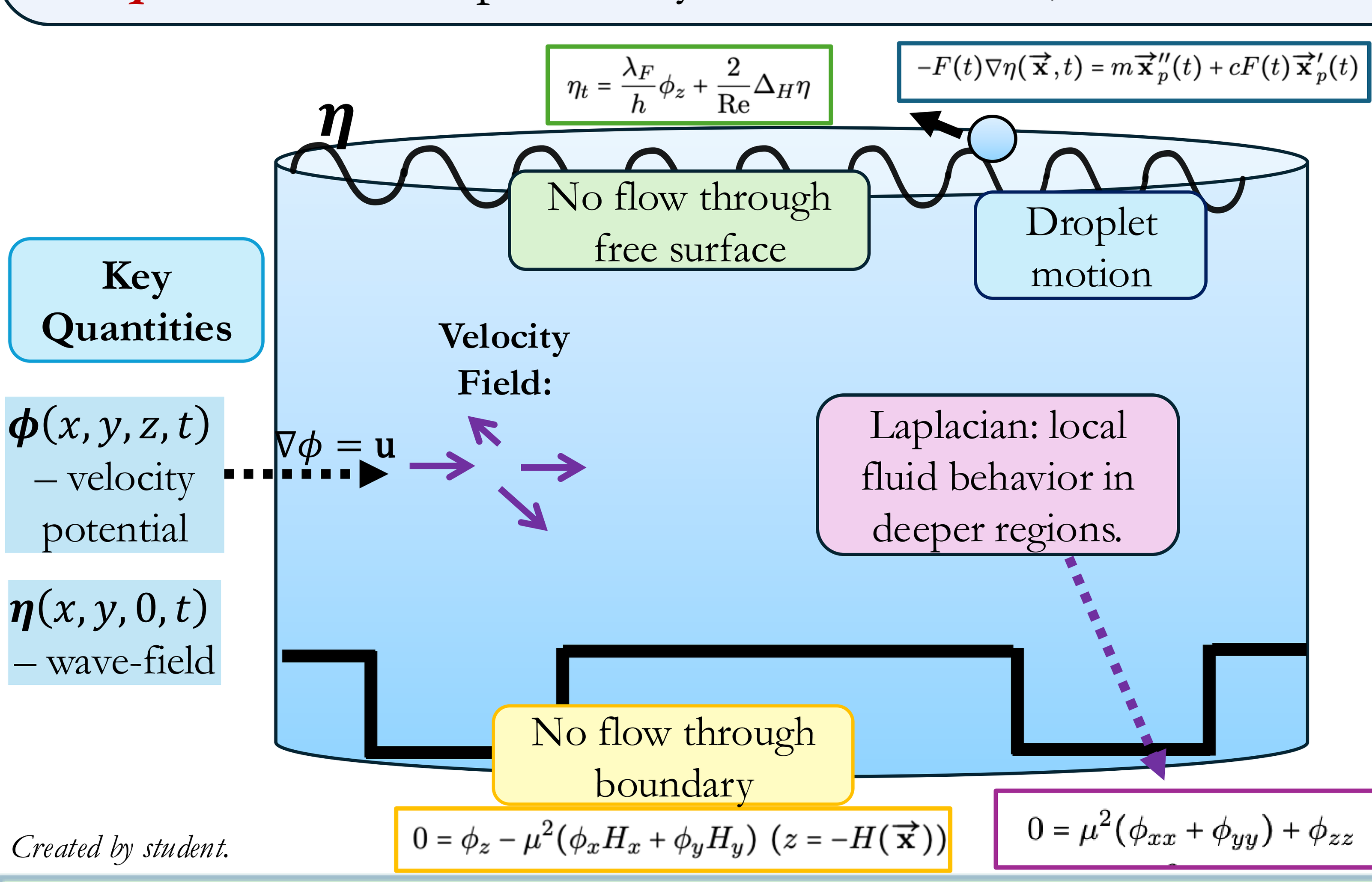
Past Research [5] → Our extensions (3D)

We **synthesize theory, simulations, and experiments** to analyze droplet dynamics and tunneling in 3D.

Theoretical Analysis

Goal: Develop the theory of **3D variable-topography droplet dynamics** while retaining **computational feasibility.**

We decompose the **Navier-Stokes Equations** (for 3D, **vector-valued fields**) to obtain an **elliptic PDE system** in two **scalar-valued** fields and use a ***Dirichlet-to-Neumann operator*** to collapse this system into **linear, 2D ODE's**.



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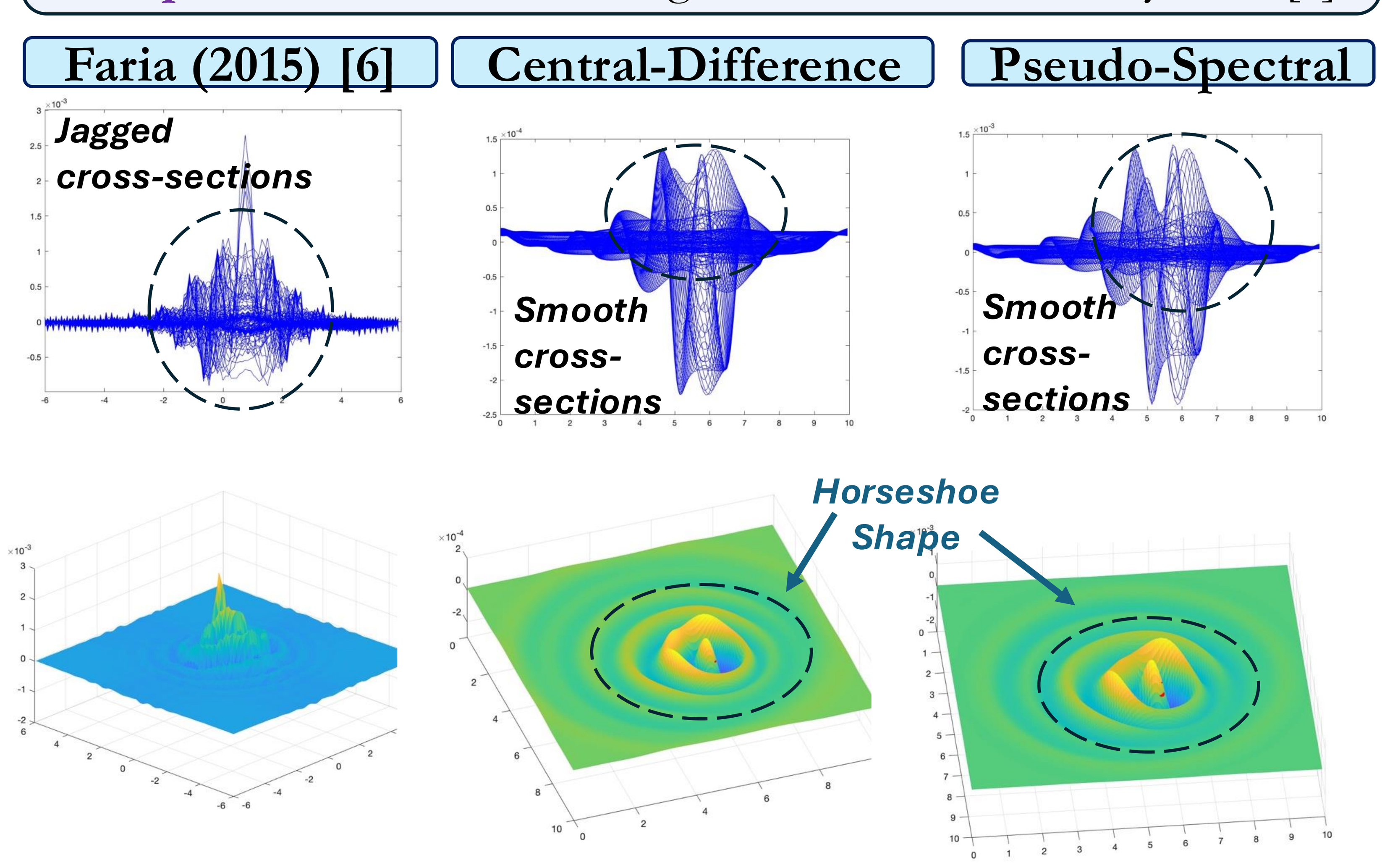
Physical illustration of the elliptic PDE system for variable-topography droplet dynamics.

Numerical Simulations

- We implement our theoretical framework for droplet dynamics using a comprehensive, **ground-up numerical model.**
- We ***time-march*** the wave-field using both: (i) **second-order central-difference** and (ii) **Fourier pseudo-spectral** schemes.

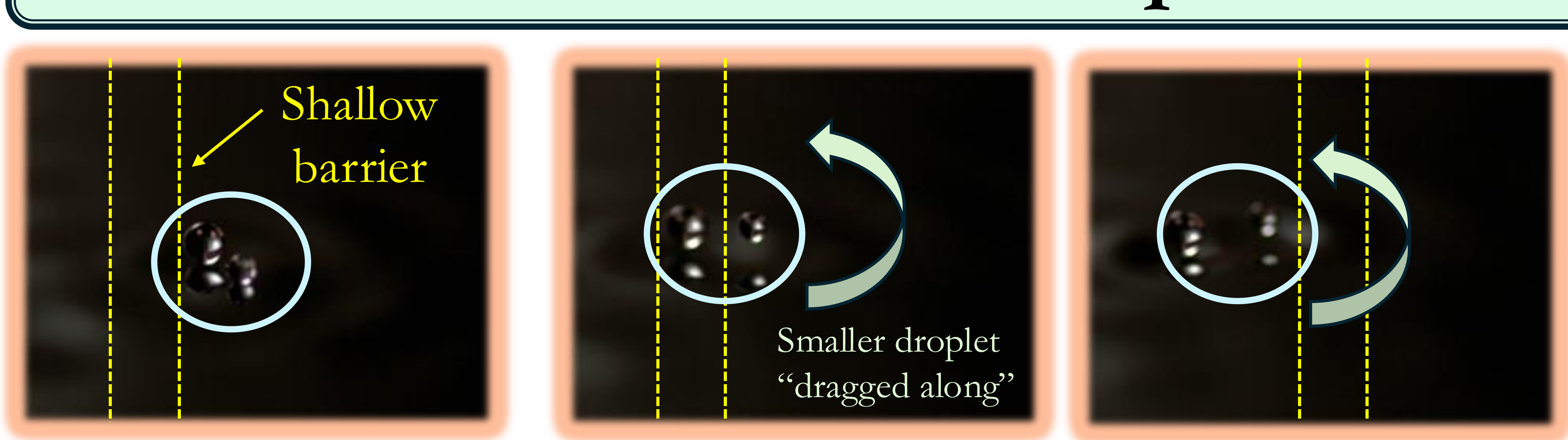
Our models yield wave-field profiles that are **smoother** and **closer to experiment** than the existing state-of-the-art code by Faria [6].



Graphs created by student.

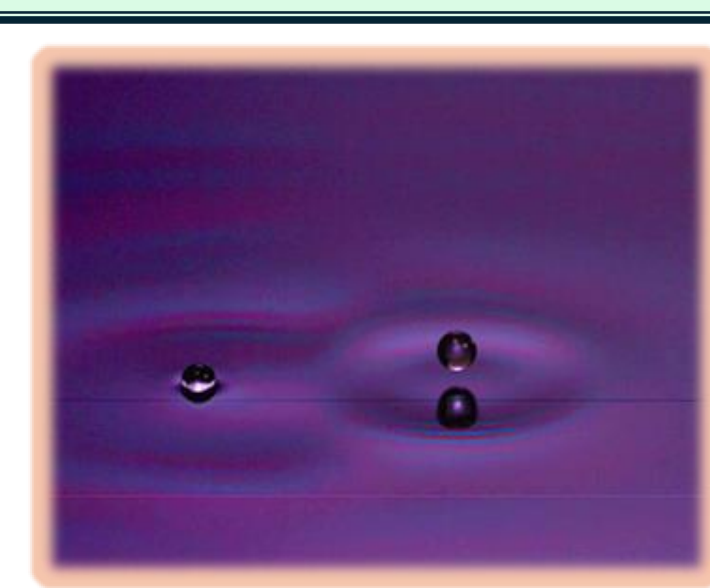
Wave-fields for walking droplets (bottom), alongside cross-sectional profiles (top).

Experimental Results

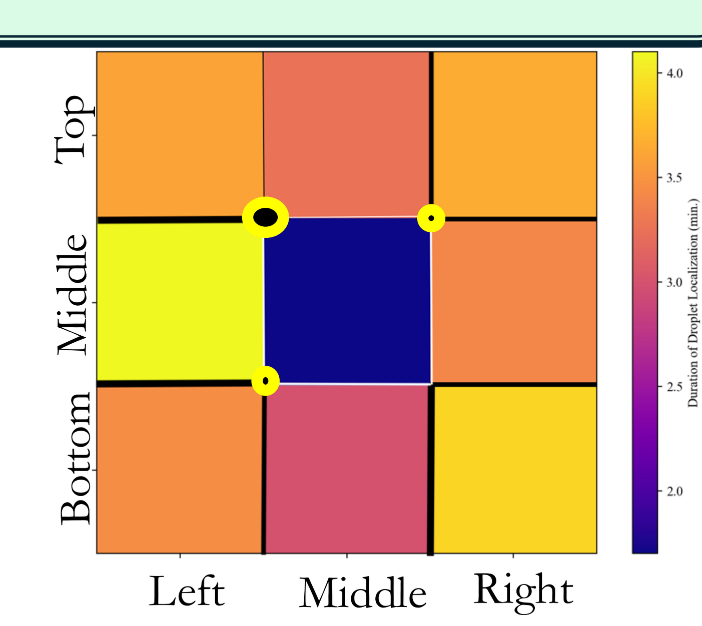


Images created by student using high-speed camera.

We observed that interactions between droplets of differing sizes establish **cooperative tunneling**, which generates ***higher crossing probabilities*** than those for a single droplet of any size.



Bouncing droplets exhibit ***non-local wavefield interactions.***



Created by student using Python and Matplotlib.

Droplets develop **angular momentum** while tunneling.

Key Contributions

- Our **theoretical formulation** of droplet dynamics offers a framework for studies of **shallow-water coastal dynamics.** [7]

- We used our theoretical description to develop the most accurate known ***model for droplet dynamics***.
- We experimentally characterized 2D droplet tunneling.

Future Work

- Compare ***tunneling probabilities*** from theory and experiment.
- Formally ***rationalize cooperative tunneling.***
- Utilize ***computer clusters*** to accelerate time-evolution process in numerical model.

References

1) Y. Couder, E. Fort, C.-H. Gautier, et al. From bouncing to floating: noncoalescence of drops on a fluid bath. *Physical review letters*, 94(17), 2005.
2) A. U. Oza, R. R. Rosales, and J. W. Bush. A trajectory equation for walking droplets: hydrodynamic pilot-wave theory. *Journal of Fluid Mechanics*, 737: pp. 552–570, 2013.
3) J. Walker. Drops of liquid can be made to float on the liquid. What enables them to do so? *Scientific American*, 238(6):pp. 151–158, 1978.
4) A. Eddi, E. Fort, F. Moisy, et al. Unpredictable tunneling of a classical wave-particle association. *Physical review letters*, 102(24), 2009.
5) A. Nachbin, P. A. Milewski, and J. W. Bush. Tunneling with a hydrodynamic pilot-wave model. *Physical Review Fluids*, 2(3), 2017.
6) L. M. Faria. A model for faraday pilot waves over variable topography. *Journal of Fluid Mechanics*, 811:pp. 51–66, 2017.
7) [Graphic Source] "Sea Wave Hd Transparent." *Pngtree*, pngtree.com/freepng/sketch-style-pencil-drawing-black-wave-seascape-sea-waves_6655974.html.