



Figure 1: [3] Nowicka, A. (2026). *Tectonic earthquakes* [Illustration]. In Caltech Science Exchange, *What happens during an earthquake?* <https://scienceexchange.caltech.edu/topics/earthquakes/what-causes-earthquakes>

Listening in the Right Places: Game Theory and Optimization For Earthquake Sensor Placement

Problem

When Nash bargaining was introduced in 1950, it provided a mathematical framework for predicting how different parties might reach an agreement [1]. Since then, Nash bargaining has mainly been used in economics, but more recently it has been applied to other fields, such as international affairs, social welfare, and more [2]. A weighted sum is a commonly known method for optimization. Each value in the sum is assigned a weight, allowing one to decide the relative importance of each factor. Weighted sums appear in statistics, data analysis, and decision-making systems. **Understanding the differences between Nash bargaining and weighted sum methods could be crucial in real-world settings.** One such case is the placement of earthquake sensors.

Earthquakes happen when the pressure between two tectonic plates builds to the point where it overcomes the friction, and this causes sections of the crust to break, releasing seismic waves [3]. After reading an article in a physics journal [4] about earthquake forecasting, I was inspired to investigate the difficulties of earthquake detection in terms of seismic sensor placement. Such mathematical approaches for earthquake sensor placement **are new and valuable tools for detecting such natural disasters as quickly as possible while ensuring installation and cost are reasonable. This is especially useful in high risk, hard to access areas that have limited resources.**

Objectives

- Create two optimization frameworks for earthquake sensor placement based on a weighted sum and Nash bargaining [1,2,5].
- Form genetic algorithms (GAs) based on the two approaches to identify the top 5 sensor locations in the Los Angeles region considering competing objectives [6,7,8]
- Generate Pareto-front plot and summary metrics comparing trade-offs.

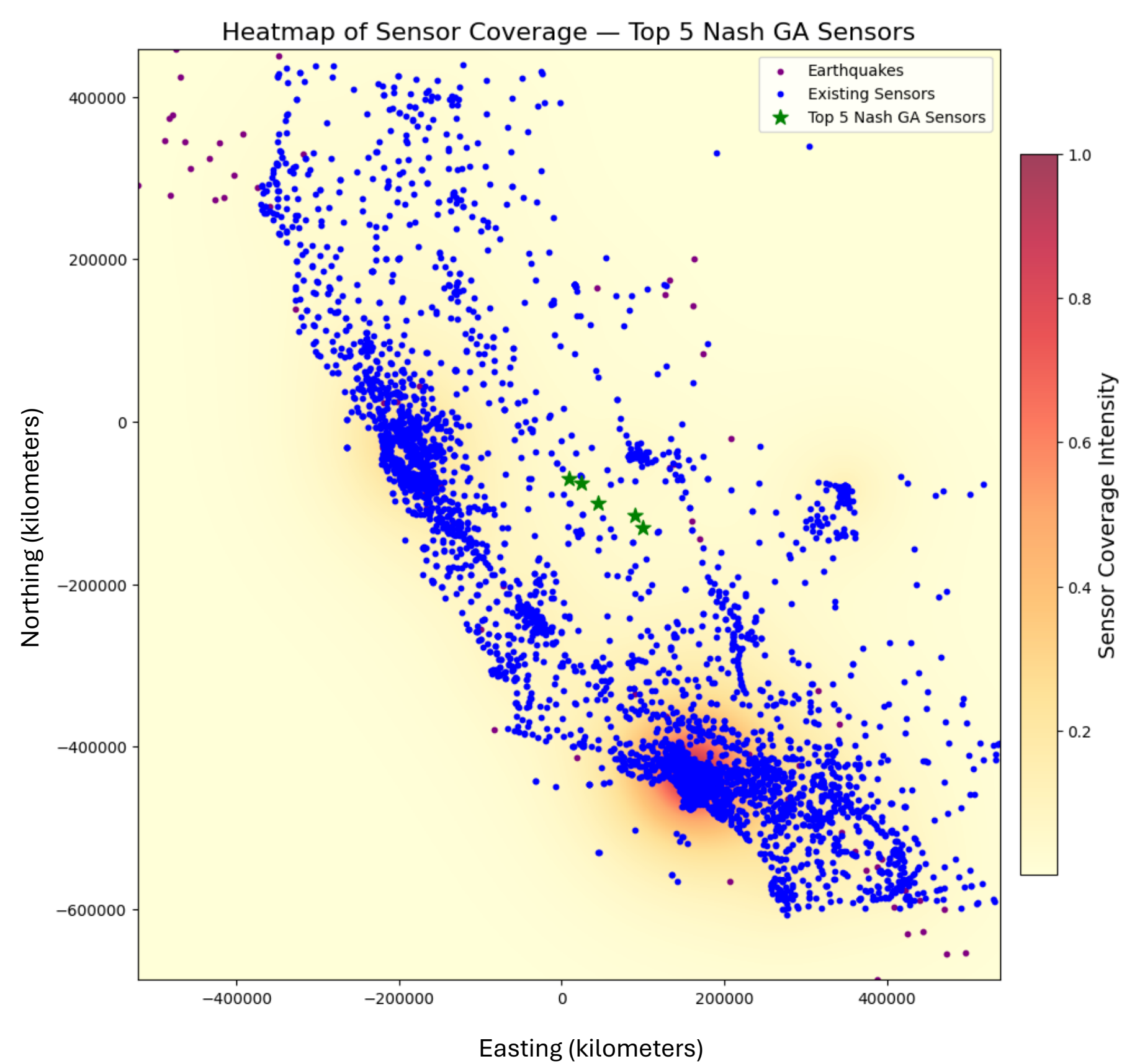


Figure 2: Heatmap sensor with suggested locations created by finalist in Python, 2026.

Procedure & Modeling

$$N(\mathbf{u}) = (u_{\text{acc}} - d_{\text{acc}})(u_{\text{diff}} - d_{\text{diff}}) \quad (1)$$

Variables: $N(\mathbf{u})$ is the Nash bargaining objective value; $u_{\text{acc}}, u_{\text{diff}}$ are the accuracy/difficulty utilities; $d_{\text{acc}}, d_{\text{diff}}$ are the corresponding disagreement points. The accuracy and difficulty utilities are each reduced by their respective disagreement points (minimum acceptable utility levels).

$$u_{\text{acc}} = \frac{\sum_{(x,y) \in \mathcal{G}} c_{\text{new}}(x,y) - \sum_{(x,y) \in \mathcal{G}} c_{\text{old}}(x,y)}{\sum_{(x,y) \in \mathcal{G}} c_{\text{old}}(x,y)} \quad (3)$$

Variables: u_{acc} is the continuous-field accuracy improvement; $\mathcal{G}$ is the evaluation grid; $c_{\text{old}}(x,y)$ and $c_{\text{new}}(x,y)$ are the continuous coverage fields for the old/new layouts. Equation (3) defines the fractional improvement in accuracy using the continuous coverage field $c(x,y)$ over the evaluation grid $\mathcal{G}$. Here, $c_{\text{old}}(x,y)$ is computed from the existing sensor set S_{cur} , and $c_{\text{new}}(x,y)$ is computed after adding the proposed sensors.

$$I(e, S) = \begin{cases} 1, & \exists s \in S \text{ such that } d(e, s) \leq R \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

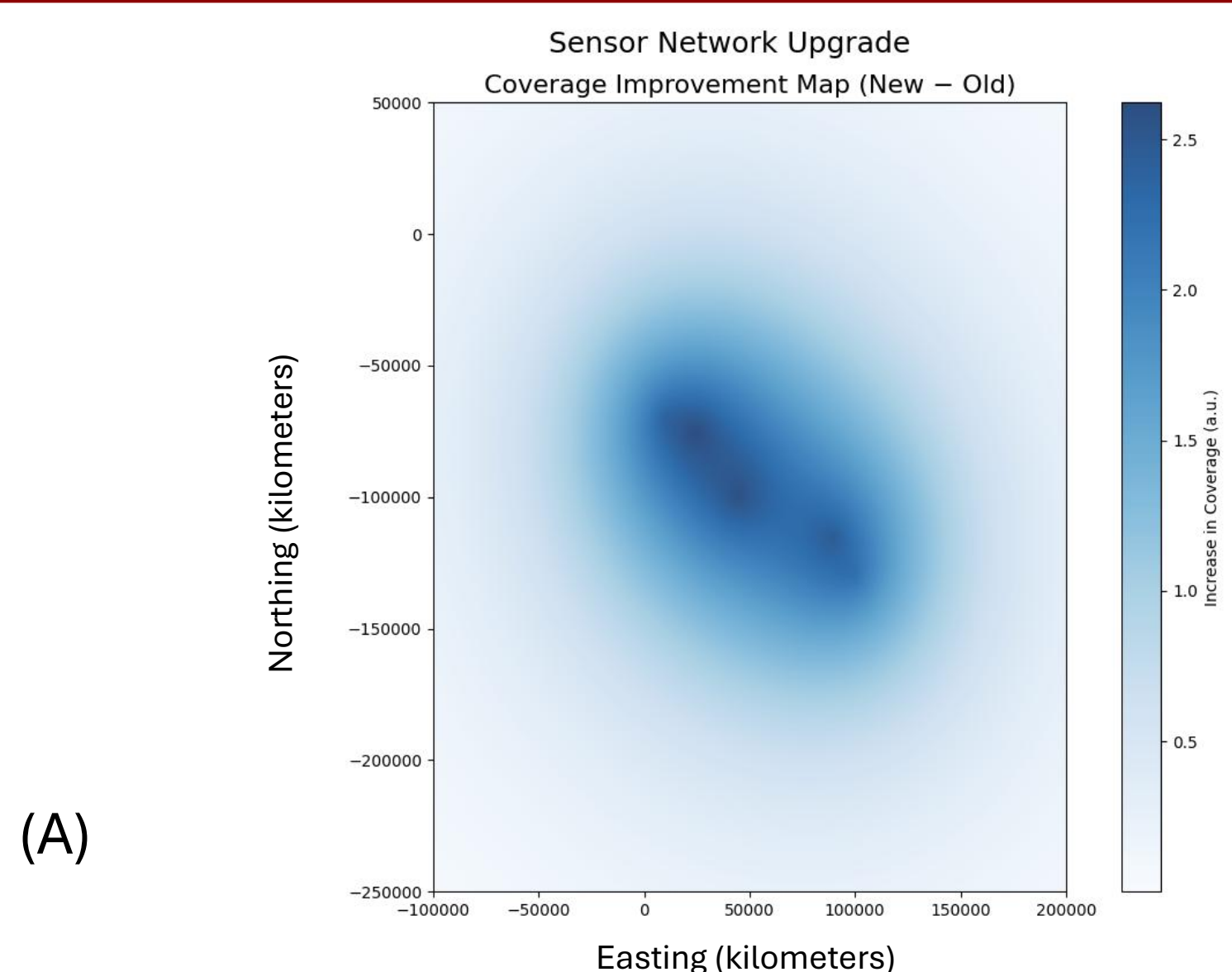
Variables: $I(e, S)$ is a binary indicator; e is an earthquake in E ; S is the sensor set; s is a sensor in S ; $d(e, s)$ is the distance between earthquake e and sensor s ; R is the coverage radius. Equation (4) defines what it means for an earthquake to be "covered." If any sensor in the set is within radius R of the earthquake, then that earthquake is considered detected; otherwise it is not.

$$\text{Coverage}(S) = \frac{1}{|E|} \sum_{e \in E} I(e, S) \quad (5)$$

Variables: $\text{Coverage}(S)$ is the fraction of earthquakes covered by sensor set S ; E is the set of earthquakes; $|E|$ is the number of earthquakes; $I(e, S)$ is the indicator from Equation (4). Equation (5) calculates the fraction of earthquakes covered by a sensor set S . For each earthquake $e \in E$, the indicator $I(e, S)$ contributes 1 if the event is covered and 0 otherwise; summing over all earthquakes and dividing by $|E|$ yields a coverage fraction.

$$\text{Acc}_{\text{cur}} = \frac{\text{Coverage}(S_{\text{cur}})}{|S_{\text{cur}}|} \quad (6)$$

Variables: Acc_{cur} is binary coverage per current sensor; S_{cur} is the current sensor set; $|S_{\text{cur}}|$ is the number of current sensors. Equation (6) is the accuracy per current sensor. The overall coverage is divided by $|S_{\text{cur}}|$ to penalize solutions that use more sensors. Although both binary earthquake coverage and continuous field improvement are referred to as measures of accuracy, only the continuous field improvement metric is used as the accuracy utility u_{acc} in the weighted-sum and Nash bargaining objectives, while binary coverage is reported separately as a descriptive performance metric.



(A)

$$W(\mathbf{u}) = w_{\text{acc}} u_{\text{acc}} + w_{\text{diff}} u_{\text{diff}} \quad (2)$$

Variables: $W(\mathbf{u})$ is the weighted-sum objective value; $w_{\text{acc}}, w_{\text{diff}}$ are the weights; $u_{\text{acc}}, u_{\text{diff}}$ are the utilities. Equation (2) is the weighted-sum objective. Each utility (accuracy and difficulty) is multiplied by a weight. The goal is to compare these two methods and determine which one better identifies balanced solutions.

$$c(x, y) = \sum_{s \in S} \exp\left(-\frac{d((x, y), s)}{D_0}\right) \quad (7)$$

Variables: $c(x, y)$ is the continuous coverage at point (x, y) ; S is the sensor set; s is a sensor in S ; D_0 is the decay distance; $d((x, y), s)$ is the distance from (x, y) to sensor s . Equation (7) defines exponential decay. For each evaluation point (x, y) , the coverage field sums contributions from every sensor $s \in S$ based on distance $d((x, y), s)$. Points nearer to sensors receive larger contributions (closer to 1), while points farther away receive smaller contributions (closer to 0).

$$c_{\text{norm}}(x, y) = \frac{c(x, y)}{\max_{x,y} c(x, y)} \quad (8)$$

Variables: $c_{\text{norm}}(x, y)$ is the normalized continuous coverage; $\max_{x,y} c(x, y)$ is the maximum continuous coverage over the evaluation domain. Equation (8) normalizes the coverage values. The maximum coverage value is used to scale all other values so that comparisons are consistent.

$$\Delta P = \frac{\sum_{(x,y) \in \mathcal{G}} c_{\text{norm,new}}(x, y) - \sum_{(x,y) \in \mathcal{G}} c_{\text{norm,old}}(x, y)}{\sum_{(x,y) \in \mathcal{G}} c_{\text{norm,old}}(x, y)} \times 100 \quad (9)$$

Variables: ΔP is the percent improvement in normalized continuous coverage; $\mathcal{G}$ is the evaluation grid; $c_{\text{norm,old}}(x, y)$ and $c_{\text{norm,new}}(x, y)$ are the normalized continuous coverage fields for the old/new layouts. Equation (9) defines the relative percent improvement in the normalized continuous coverage field after adding the sensors. If the result is positive, then (on average over $\mathcal{G}$) the continuous detection proxy has improved.

$$\text{EQScore}_i = 1 - \frac{d_i^{\text{eq}}}{\max_j d_j^{\text{eq}}} \quad (10)$$

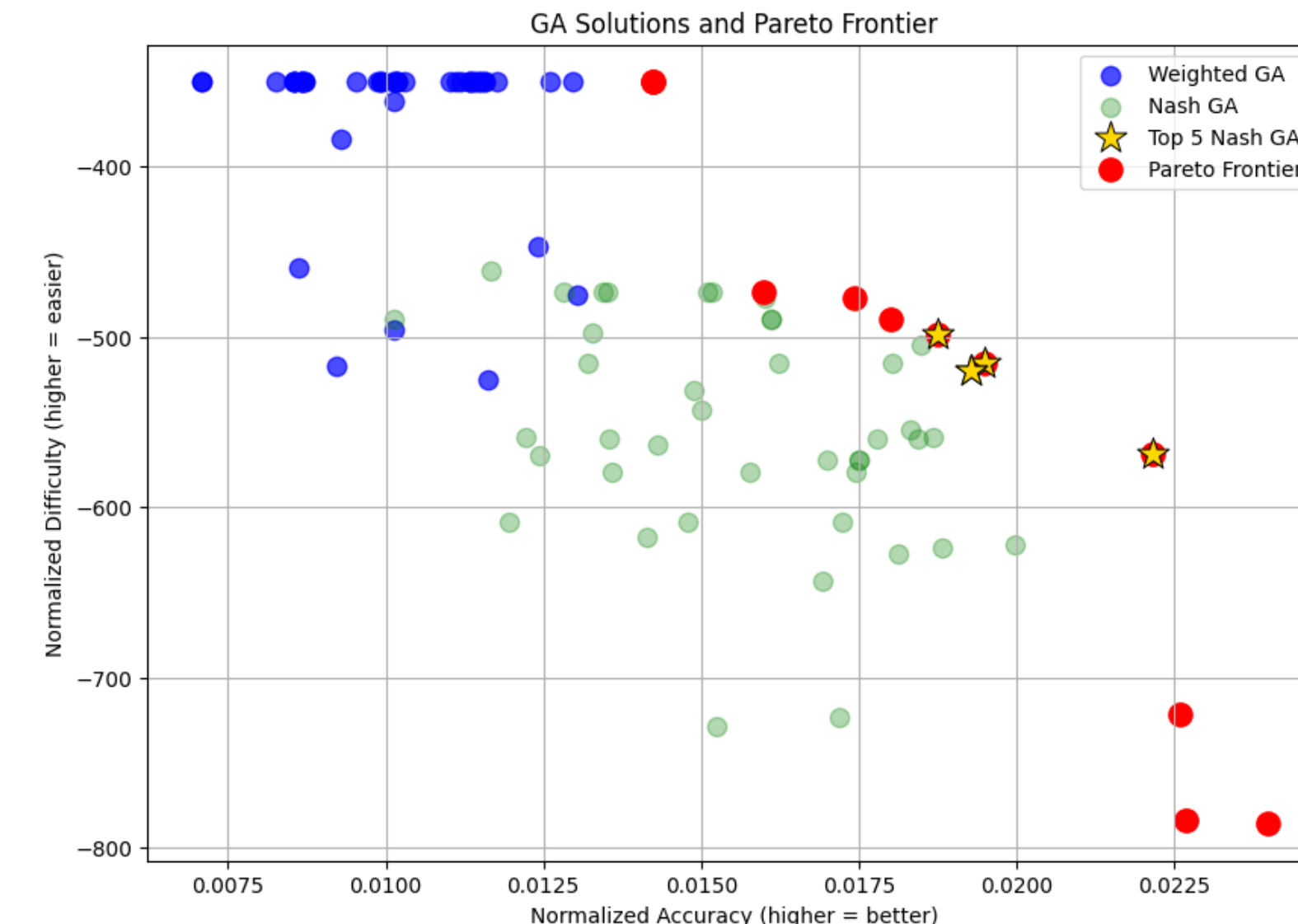
Variables: EQScore_i is the earthquake proximity score for candidate i ; d_i^{eq} is the distance from candidate i to the nearest earthquake; $\max_j d_j^{\text{eq}}$ is the maximum such distance over candidates. Equation (10) defines an earthquake proximity score. Candidate sensor locations closer to earthquake-prone areas receive scores closer to 1, while farther candidates receive scores closer to 0.

$$\text{RoadScore}_i = 1 - \frac{d_i^{\text{road}}}{\max_j d_j^{\text{road}}} \quad (11)$$

Variables: RoadScore_i is the road proximity score for candidate i ; d_i^{road} is the distance from candidate i to the nearest road; $\max_j d_j^{\text{road}}$ is the maximum such distance over candidates. Equation (11) defines a road proximity score. Candidate locations closer to roads receive scores closer to 1, reflecting lower installation and maintenance difficulty.

$$\text{LocationScore}_i = w_{\text{eq}} \text{EQScore}_i + w_{\text{road}} \text{RoadScore}_i \quad (12)$$

Variables: LocationScore_i is a combined location score for candidate i ; w_{eq} and w_{road} are weights (typically summing to 1); EQScore_i and RoadScore_i are the component scores. Equation (12) defines a weighted-sum location score that combines earthquake proximity and road proximity.



(B)

Figure 3: (A) Coverage Improvement Map with Sensor Network Upgrade; Figure 4: (B) Weighted sum and Nash bargaining GA solutions compared to the Pareto frontier. The Pareto frontier was found by comparing all GA generated sensor sets and keeping only those that could not be improved in one aspect without worsening the others. Images created by finalist using Python, 2026.

Discussion

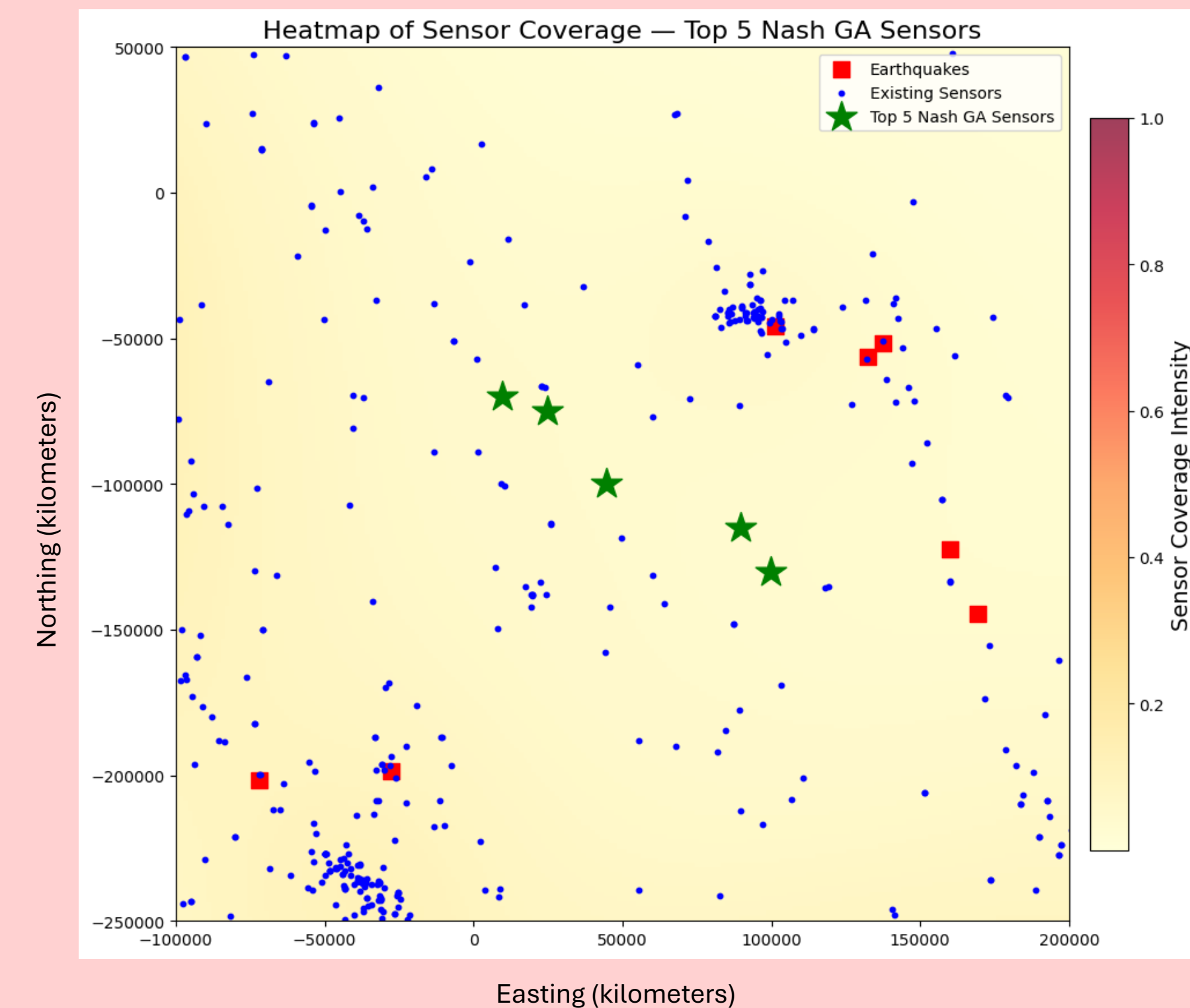


Figure 5: Map of coverage for the area of LA. Image created by finalist in Python, 2026.

Each sensor selected by the Nash GA covered ~3.% of the study area on average, compared to ~0.3% per existing sensor. After adding the five optimized sensors to the Overall, in a region around LA, the network **increased total coverage by ~15%.** Results showed Nash bargaining to be better at finding optimized sensor placement as its solutions were closer to the Pareto frontier.

Conclusion

The developed method was successful at identifying top locations that improved accuracy while also taking into consideration other limitations. This could be implemented in under-resourced areas and increase earthquake safety.

Future work could include:

- Comparing a larger quantity of low-cost sensors with a smaller number of expensive sensors and analyzing differences and similarities.
- Applying a game theoretic approach to other optimization problems or fields, such as flight planning or medicine.

References

- [1] John Nash, J. F., Jr. (1950). *The bargaining problem*. *Econometrica*, 18(2), 155–162.
- [2] Niederle, M. (n.d.). *Nash bargaining*. Stanford University. Retrieved from <https://web.stanford.edu/~niederle/Nash%20Bargaining.pdf>
- [3] Caltech Science Exchange. (2026). *What happens during an earthquake?* <https://scienceexchange.caltech.edu/topics/earthquakes/what-causes-earthquakes>
- [4] American Institute of Physics. (n.d.). *The pursuit of reliable earthquake forecasting*. *Physics Today*. <https://physicstoday.aip.org/features/the-pursuit-of-reliable-earthquake-forecasting>
- [5] Bateni, M. H., et al. (2010). The cooperative game theory foundations of network bargaining games. In *International Colloquium on Automata, Languages, and Programming*. Berlin, Heidelberg: Springer.
- [6] Pirani, M., Nekouei, E., Sandberg, H., & Johansson, K. H. (2021). A game-theoretic framework for security-aware sensor placement problem in networked control systems. *IEEE Transactions on Automatic Control*, 67(7), 3699–3706.
- [7] Callahan, J., Monogue, K., Villarreal, R., & Catanach, T. (2025). Analysis and optimization of seismic monitoring networks with Bayesian optimal experimental design. *Geophysical Journal International*, 240(3), 1802–1824.
- [8] Choi, H. L., & Lee, S. J. (2015). A potential-game approach for information-maximizing cooperative planning of sensor networks. *IEEE Transactions on Control Systems Technology*, 23(6), 2326–2335.

Isabel Rossi