Diffusion-Only Water Transport and How To Modulate Its Speed

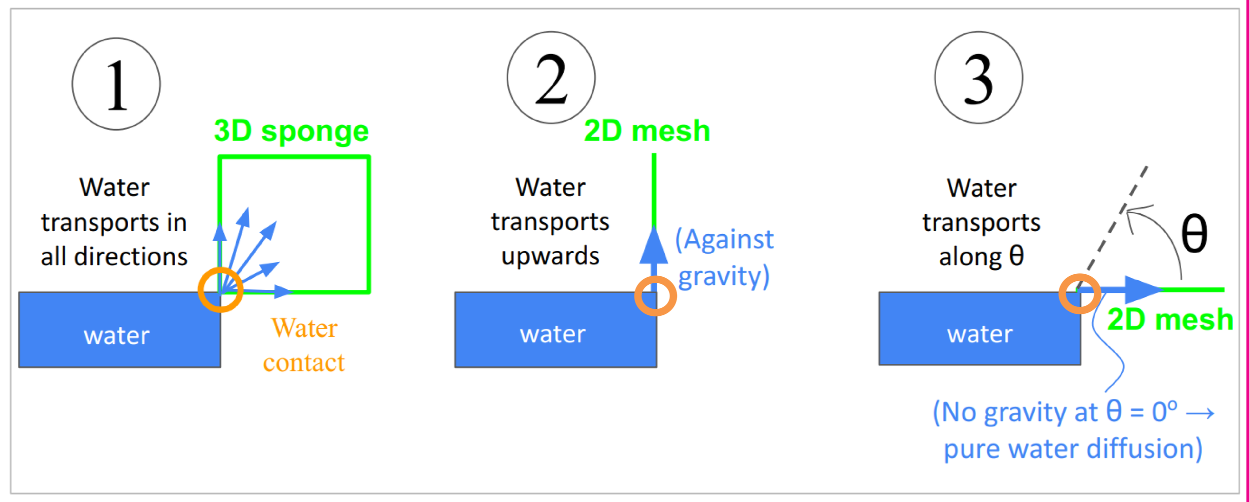


Figure 1: How 3D water transport is simplified to 2D diffusion (at θ = 0°).

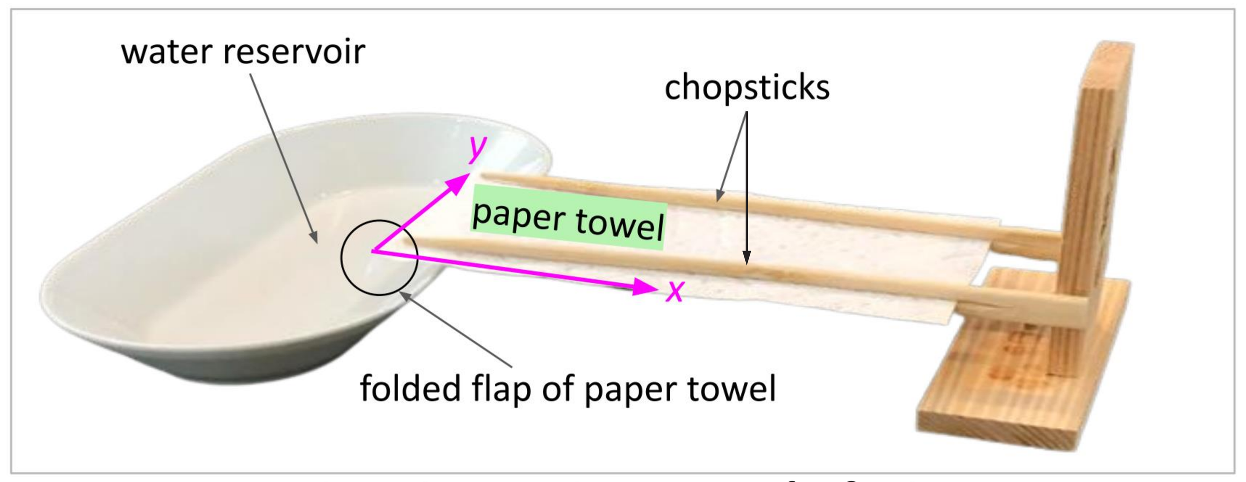


Figure 2a: Experiment setup for θ = 0°.

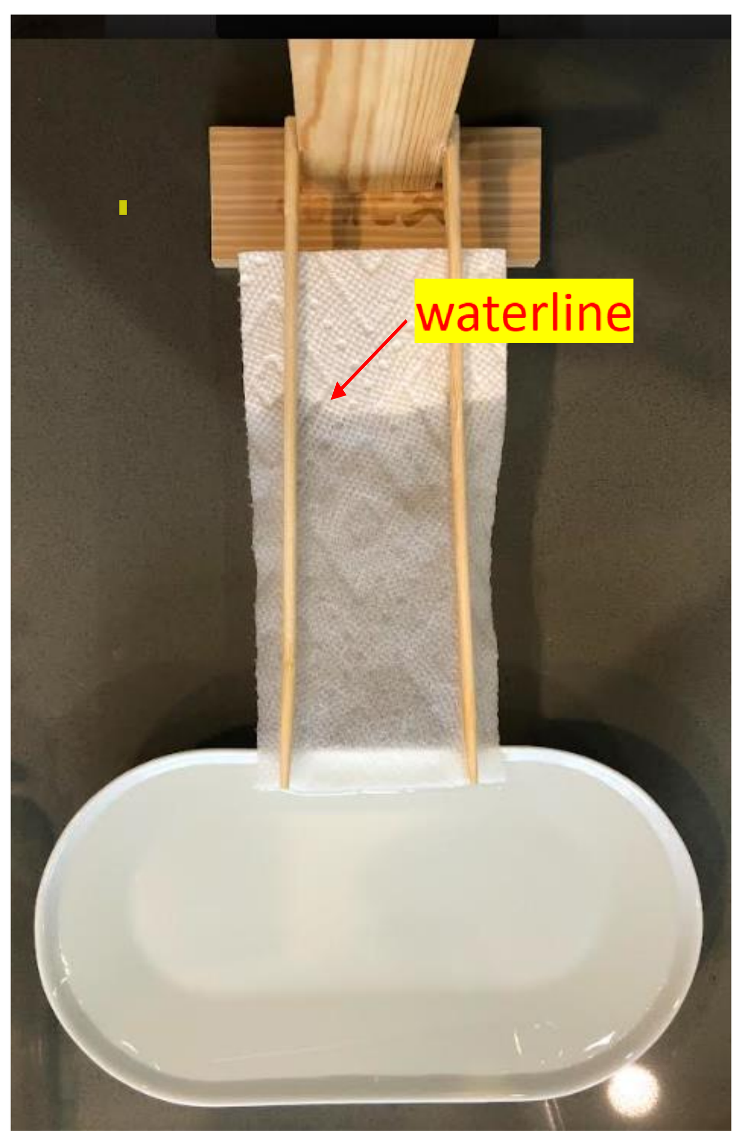


Figure 2b: Experiment for θ = 0°.


Figure 2c: Measuring waterlines vs. time.

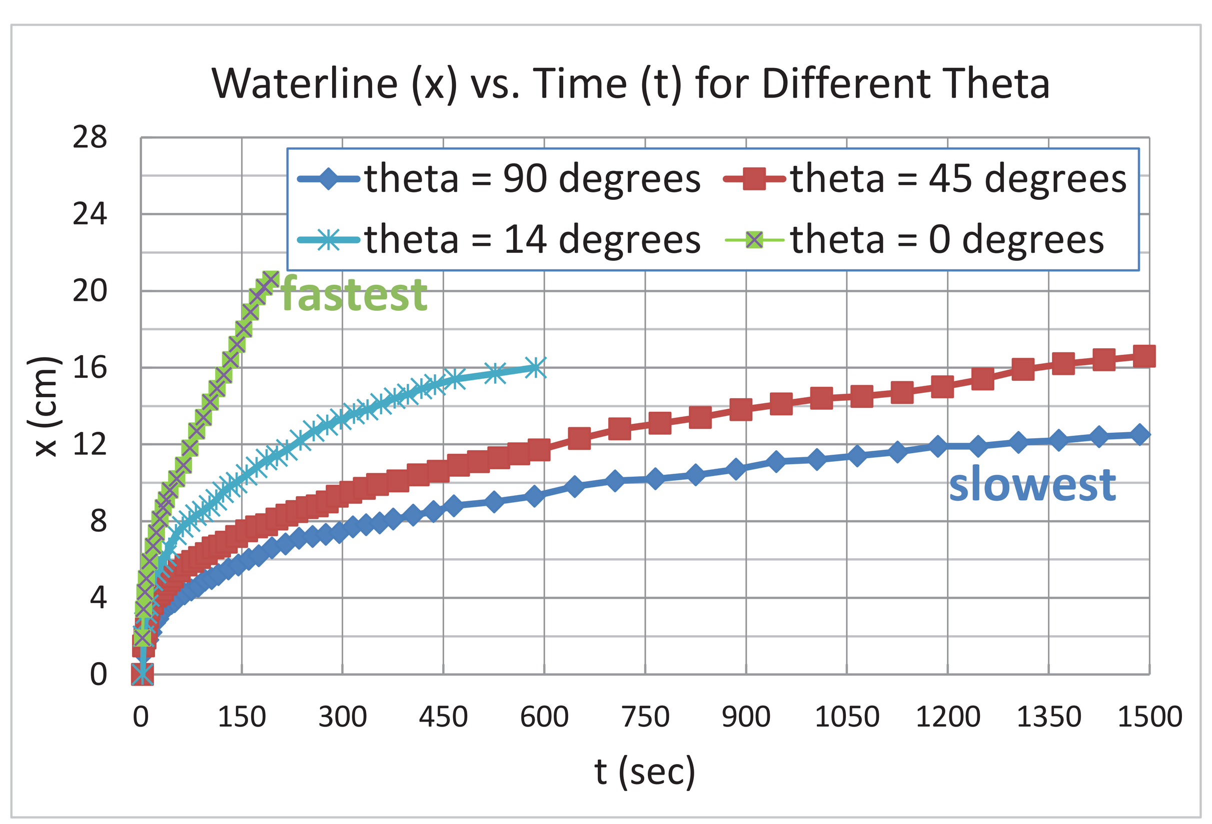


Figure 3: Measured waterline data for different θ's.

Summary

1. Water transport simplified to diffusion-only.
2. Identified erfc(x, t) model that fits water diffusion-only data.
3. Determined water diffusivity and its temperature dependence using data and model.
4. Designed shape to speed up diffusion by 19% (with respect to control).
5. Simulation of tapering-inwards boundary exhibits faster diffusion, supporting 4.

1) Flux eqn.

$$F(x) = -D\frac{\partial C}{\partial x}$$

2) Continuity eqn.

$$\frac{\partial C}{\partial t} = -\frac{\partial F}{\partial x} = D\frac{\partial^2 C}{\partial x^2}$$

3) Init. & bound. condi.

C(x, 0) = 0; C(∞, t) = 0;
C(0, t) = C_s (soaked water concentration)

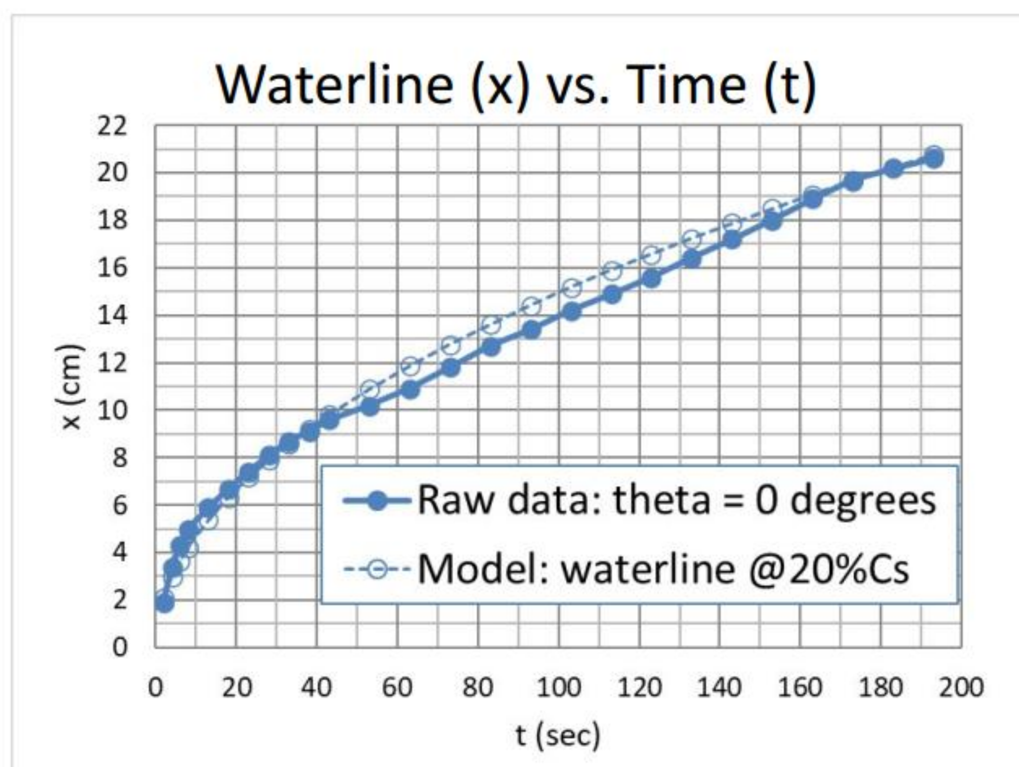
4) Solution of Concen.

$$C(x,t) = C_S \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

(Grove, 1967)

5) Waterline data and model

Model: $x|_{\text{waterline}} = 2\sqrt{Dt} \times \operatorname{erfc}^{-1}(20\%)$



6) Extraction of diffusivity, *D*

Plugging C(x, t) and D back into 1):

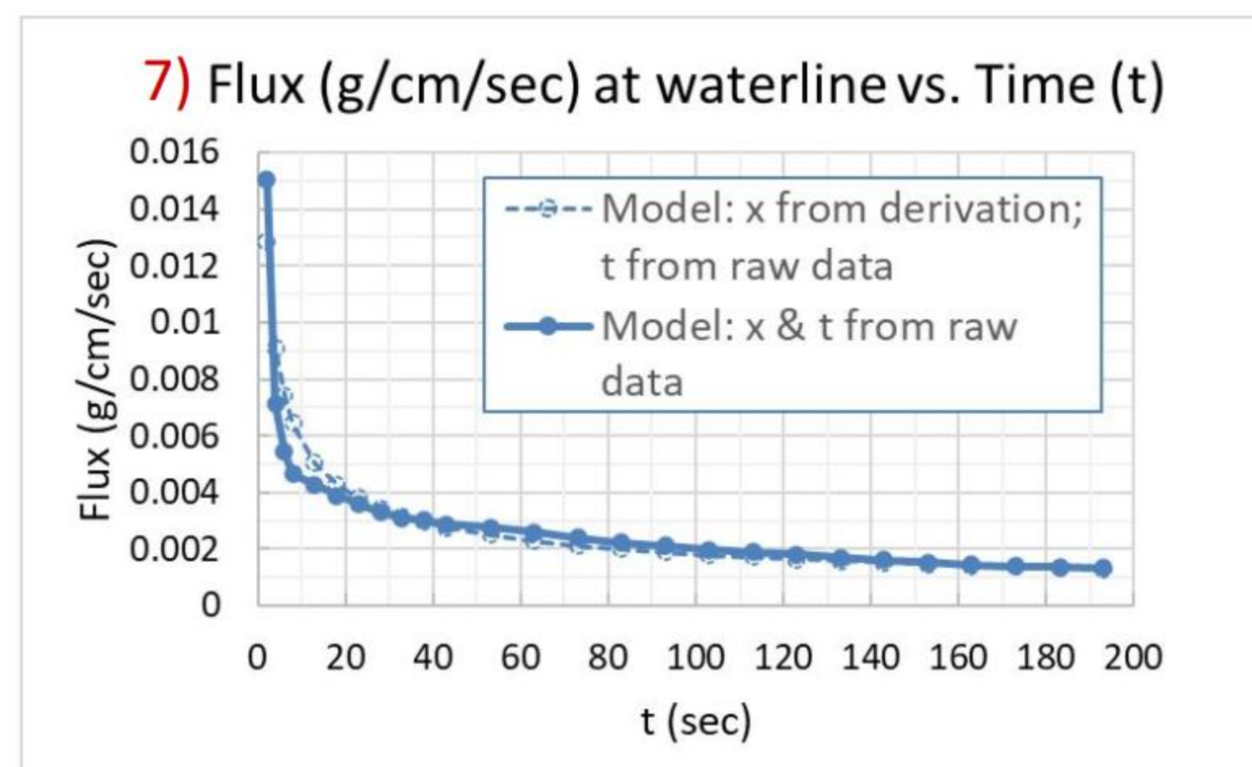
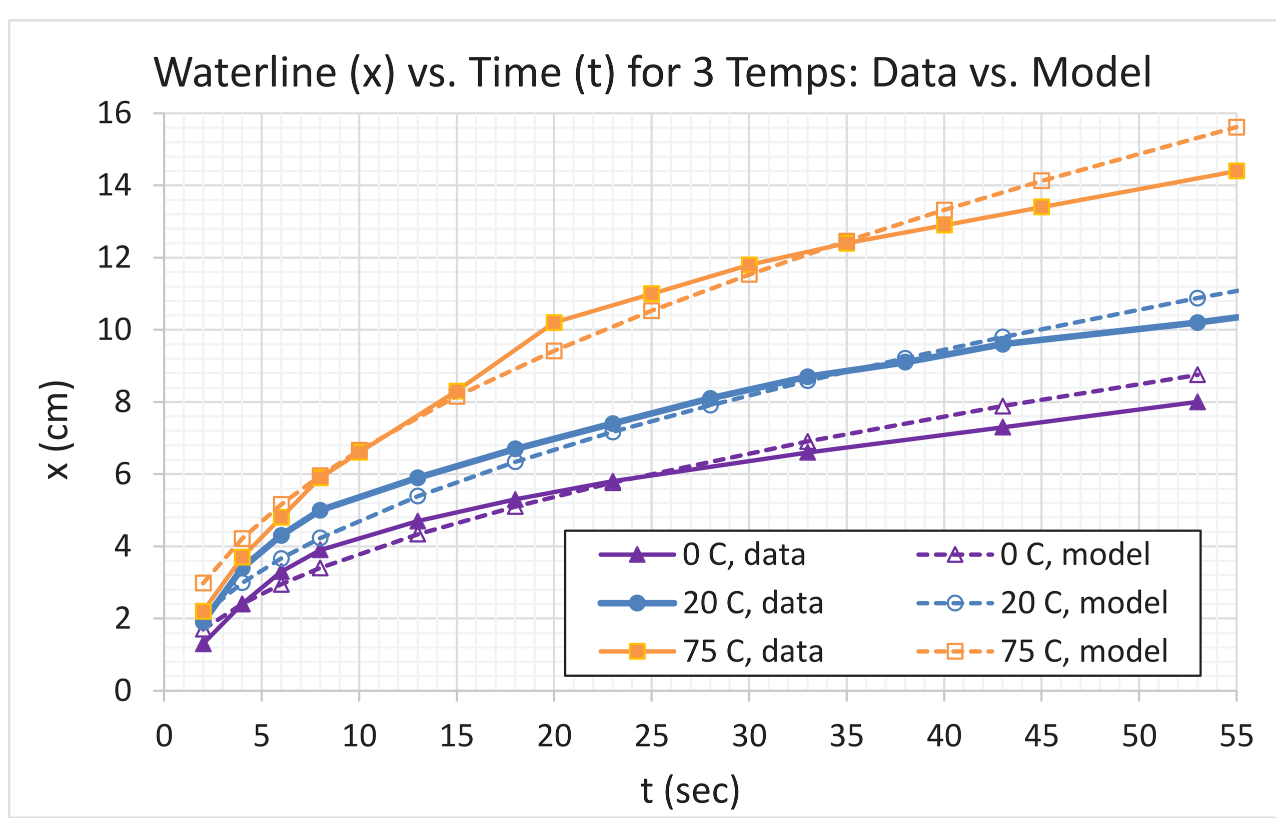
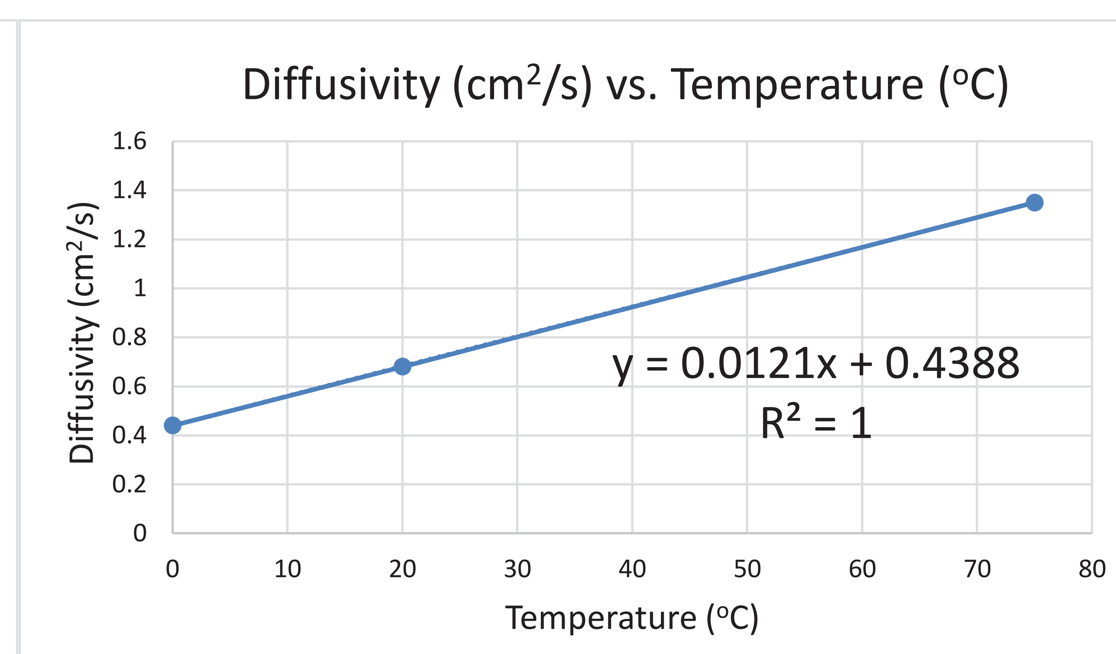


Figure 4: Development of 2D analytical water diffusion model in 7 steps. Note in 5), C@waterline/Cs = 20% is obtained through weight comparison – weight of a paper strip along y at waterline vs. that of soaked paper. Weight of paper strip itself is negligible.

Symbol	Meaning	Unit
C(x,t)	Water concentration (along a 2D mesh)	g/cm²
D	Water diffusivity along the 2D mesh	cm²/sec
F(x) or F(x,t)	Water flux	g/cm/sec





Thermal Diffusivity	Water Diffusivity along paper towel
$\frac{\text{thermal conductivity}}{\text{material density} \times \text{heat capacity}}$	$\propto$ *temperature*

Figure 5: Left – extracted diffusivity. Right – its positive temperature dependence.
(Some material's thermal conductivity is proportional to temperature).

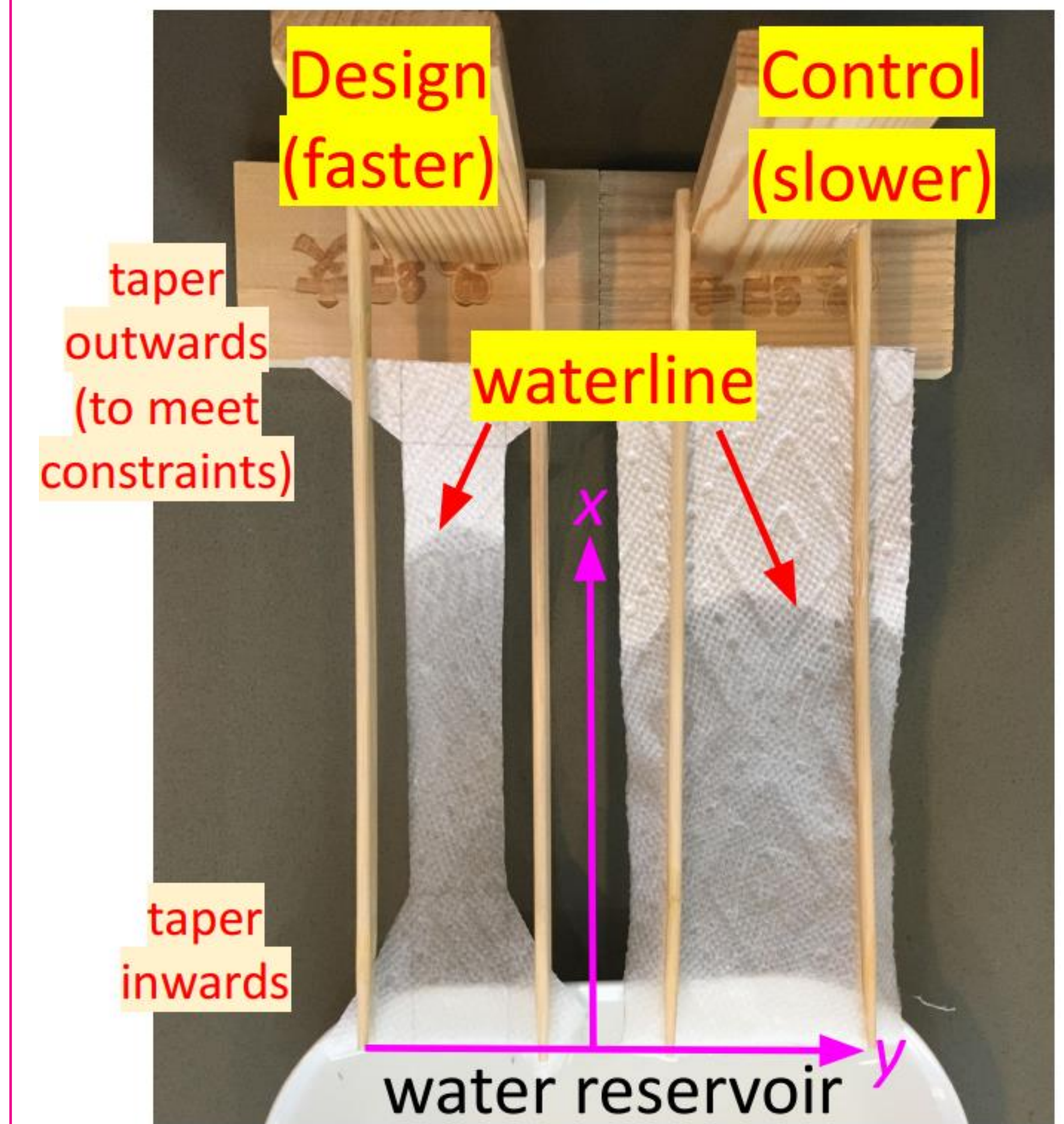


Figure 6: Paper towel shape design vs. control. Design is 19% faster. The constraint was identical paper width for source and destination.

Why is tapering-inward design faster?

Water hits tapering-inward boundaries first, accumulating higher concentration → stronger flux, faster diffusion.

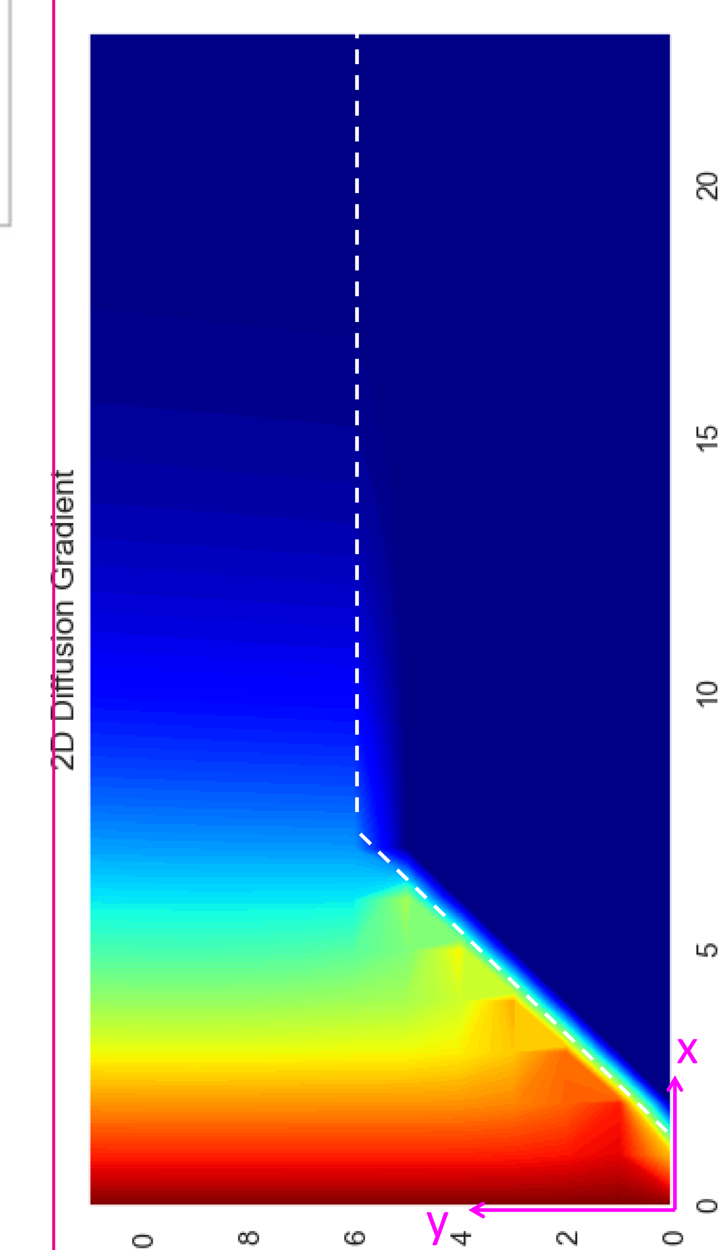


Figure 7: Simulation of 2D diffusion. Water accumulation at tapering-inwards boundary enhances diffusion speed.

Warm/cold colors mean high/low water concentration, respectively.

Bound. Condi.:
- i) x = 0: water reservoir
- ii) x = 32: dry
- iii) White dash: water accumulation

Numerical analysis based on Gerald & Wheatley, 1994.