**Summary**

1. Water transport simplified to diffusion-only.
2. Identified $erfc(x, t)$ model that fits water diffusion-only data.
3. Determined water diffusivity and its temperature dependence using data and model.
4. Designed shape to speed up diffusion by 19% (with respect to control).

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**Figure 1:** How 3D water transport is simplified to 2D diffusion (at $\theta = 0^\circ$).

**Figure 2a:** Experiment setup for $\theta = 0^\circ$.

**Figure 2b:** Experiment for $\theta = 0^\circ$.

**Figure 2c:** Measuring waterlines vs. time.

**Figure 3:** Measured waterline data for different $\theta$’s.

**Figure 4:** Development of 2D analytical water diffusion model in 7 steps.

Note in 5), $C@waterline/C_{s} = 20\%$ is obtained through weight comparison – weight of a paper strip along $y$ at waterline vs. that of soaked paper. Weight of paper strip itself is negligible.

**Figure 5:** Left – extracted diffusivity. Right – its positive temperature dependence. (Some material’s thermal conductivity is proportional to temperature).

**Figure 6:** Paper towel shape design vs. control. Design is 19% faster. The constraint was identical paper width for source and destination.

**Figure 7:** Simulation of 2D diffusion. Water accumulation at tapering-inwards boundary enhances diffusion speed.

Warm/cold colors mean high/low water concentration, respectively.

**Table:**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(x,t)$</td>
<td>Water concentration (along a 2D mesh)</td>
<td>$g/cm^2$</td>
</tr>
<tr>
<td>$D$</td>
<td>Water diffusivity along the 2D mesh</td>
<td>$cm^2/sec$</td>
</tr>
<tr>
<td>$F(x)$ or $F(x,t)$</td>
<td>Water flux</td>
<td>$g/cm/sec$</td>
</tr>
</tbody>
</table>

**Numerical analysis based on Gerald & Wheatley, 1994.**

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**Design (faster)**

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**Control (slower)**

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**Why is tapering-inward design faster?**

Water hits tapering-inward boundaries first, accumulating higher concentration → stronger flux, faster diffusion.

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**Figure 8:** Paper towel shape design vs. control. Design is 19% faster. The constraint was identical paper width for source and destination.

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**Figure 9:** Simulation of 2D diffusion. Water accumulation at tapering-inwards boundary enhances diffusion speed.

Warm/cold colors mean high/low water concentration, respectively.

Bound. Condi.:

i) $x = 0$: water reservoir
ii) $x = 32$: dry
iii) White dash: water accumulation

**Numerical analysis based on Gerald & Wheatley, 1994.**