The Stability of Solar Flares

Question

How do different changes in the state (density, velocity, pressure, or magnetic field) affect the Magnetic Flux Rope (MFR)?

Does one perturbation affect more than another?

Hypothesis

I hypothesize that density and velocity perturbations will have the greatest effect on the Magnetic Flux Rope. A change in density will change the total mass of the MFR and a change in the velocity will push plasma outwards also changing the total mass.

Background

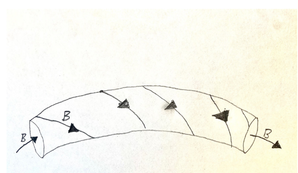
- **The Sun**

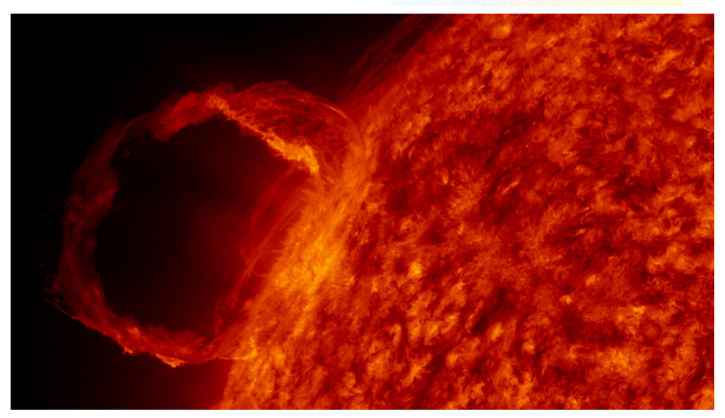
The sun is made of plasma, gas that is so hot that its atoms are separated into positive ions and negative electrons. The relative motion of the ions and electrons generates magnetic fields which direct plasma flow throughout the sun. On the outermost layer of the sun, the corona, these magnetic fields sometimes break from the surface of the sun, thus producing solar flares which can develop into Coronal Mass Ejections (CMEs).

Why should we care? Well, CMEs are big eruptions that spew tons and tons of plasma outward. When a CME hits Earth, a geomagnetic storm wreaks havoc on electrical equipments, disrupts communications, and even endanger lives. The last CME to hit Earth occurred in 2012 and caused mass electrical failures. To mitigate the damage caused by CMEs, we need to be able to model and predict when a solar flare can develop into a CME.

- **Magnetic Flux Rope (MFR)**

MFR is a model for solar flares [2], which is characterized by twisting magnetic field lines, as shown in the diagram on right. The actual solar flare, in the photo, shows the helix-like structure and demonstrates that Magnetic Flux Rope is a good model for a solar flare. Solar flares and MFRs are an active research area and there is a lot we still don't know.





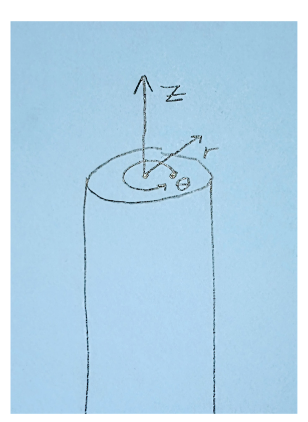
[3] NASA, *Prominence from the Solar Dynamics*

- **Stability**

The stability of a state can be understood by a marble in a bowl. If you put the marble in the center of the bowl and hit it, it will eventually end up back at the center of the bowl, stationary. If you flip the bowl over and place the marble in the exact center so that it doesn't fall, when you slightly hit it, it will fall off the bowl and will not return to the center. The stability of the marble is whether or not it returns to the initial point, whether the bowl is right side up or upside down. Analogous to a marble we can test whether an MFR is stable or not by perturbing it. If it returns to its initial equilibrium state, that means that MFR is a stable structure. If the perturbation grows and disrupts the entire Magnetic Flux Rope, MFR is unstable.

- **Coordinates**

Instead of Cartesian coordinates (x, y, z), I used cylindrical coordinates (r, θ, z), which is more natural for describing a cylindrical domain. In cylindrical coordinates, r is the radius, so the center of the MFR is r=0. Next, we have θ (theta). θ is perpendicular to r and is the angular or azimuthal position of a point. And z is the height of the point in the cylinder.



- **Magnetohydrodynamics (MHD)**

Magnetohydrodynamics (MHD) is the study of ionized gas, which is plasma. Using the laws of MHD is needed to simulate a Magnetic Flux Rope.

To model an MFR, I am going to use the equations of ideal magnetohydrodynamics, meaning there is no friction between the particles of the plasma and there is no electrical resistance in the plasma. To understand some of the formulas in ideal MHD, we need to define some variables.

ρ ("rho") is the mass density of the plasma (mass per volume)

v is the fluid velocity (movement of the ions in the plasma)

p is the pressure

E is the electric field vector (the direction and strength of the electric field)

B is the magnetic field vector (the direction and strength of the magnetic field)

J is the electrical current vector (the relative movement of the positive ions and the negative electrons)

Governing Equations

- **Full 3D Equations** [4]
  - Conservation of mass: $\frac{d\rho}{dt} = -\rho\nabla\cdot\mathrm{v}$
  - Conservation of momentum: $\rho\frac{d\mathrm{v}}{dt} = -\nabla p + \mathrm{J}\times\mathrm{B}$
  - Conservation of energy: $\rho\frac{de}{dt} = -p\nabla\cdot\mathrm{v}$
  - Faraday's law: $\nabla\times\mathrm{E} = -\frac{\partial\mathrm{B}}{\partial t}$
  - Ampère's law: $\nabla\times\mathrm{B} = \mu_0\mathrm{J}$
  - Ohm's law: $\mathrm{E} + \mathrm{v}\times\mathrm{B} = 0$

The MFR is a steady-state equilibrium solution to the above equations. That means that the MFR does not change with time. This is not true for a Magnetic Flux Rope *with a perturbation*, since it is not at equilibrium anymore. I obtained the full set of time evolution equations of magnetohydrodynamics in cylindrical coordinates from a paper [5]. Then, using my assumptions, I formed the final, simplified equations.

- **Assumptions**
  - The perturbed MFR has cylindrical symmetry ⟶ 1D equations
  - The magnetic field in the r direction has to be zero
  - r is moved inside the time derivatives to have the equations in the conservative form

- **Simplified 1D Equations**

The following equations were derived by referring to the paper [5], based on my assumptions. These are the final equations that are in my Python program.

- The conservation of mass gives the density: $\frac{\partial}{\partial t}(r\rho) + \frac{\partial}{\partial r}(r\rho v_r) = 0$
- The conservation of momentum gives the velocity: $\frac{\partial}{\partial t}(r\rho v_r) + \frac{\partial}{\partial r}\left(r\left(\rho v_r^2 + p\right)\right) = \frac{-B_\theta^2}{\mu_0} + p$
- The conservation of energy gives the energy: $\frac{\partial}{\partial t}(r\rho E) + \frac{\partial}{\partial r}[r(\rho E + P^*)v_r] = 0$

with $P^* = p + \frac{B_z^2 + B_\theta^2}{2\mu_0}$

- Faraday's, Ampère's, and Ohm's Law together give the equations for the magnetic field: one for the magnetic field in the z direction and the other for the θ component.

$$\frac{\partial}{\partial t}(rB_z) + \frac{\partial}{\partial r}(rv_rB_z) = 0$$

$$\frac{\partial}{\partial t}B_\theta + \frac{\partial}{\partial r}(v_rB_\theta) = 0$$

Results of Magnetic Flux Rope Perturbations

MFR Steady-State Solution

MFRs are a steady-state solution to the above equations. The exact solution has been found to be the following [4]. Here r is the radius of the point and $\bar{r}$ is the radius of the MFR.

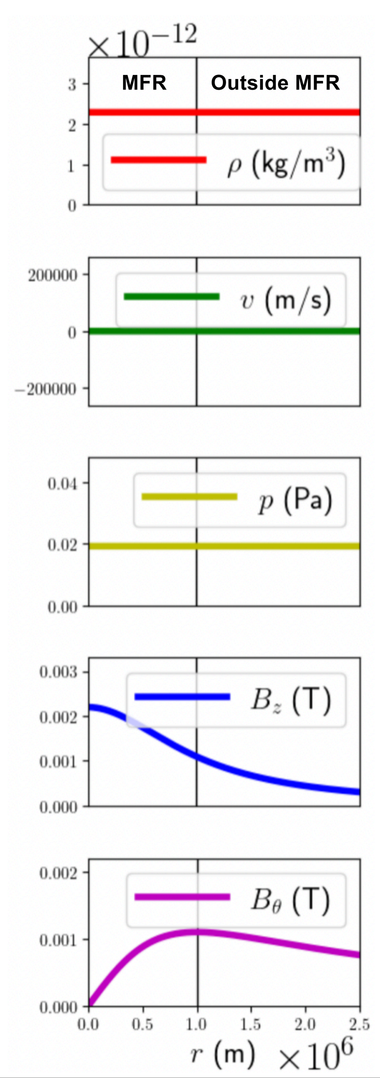
Density
- $\rho = \rho_0$

Radial Velocity
- $v_r = 0$

Pressure
- $p = p_0$

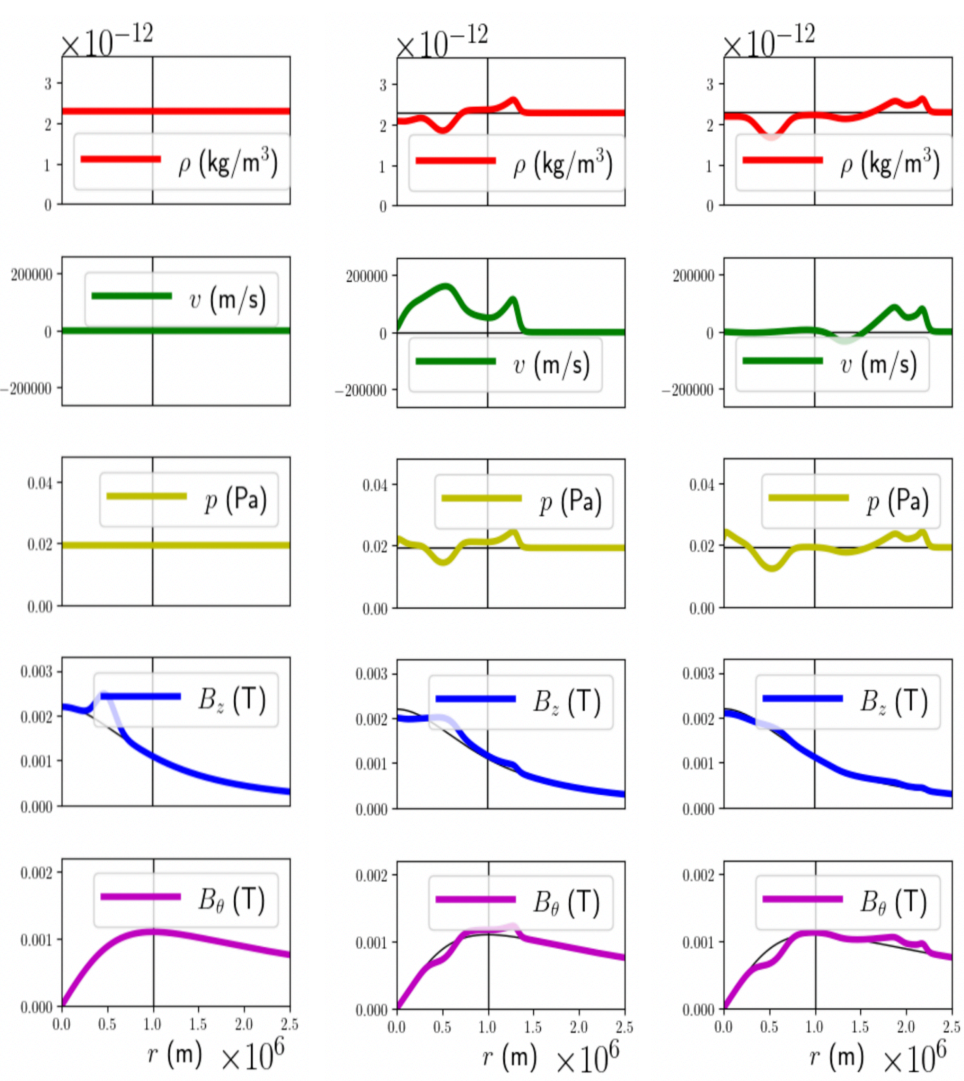
Axial Magnetic Field
- $B_z = \frac{B_0}{1 + (r/\bar{r})^2}$

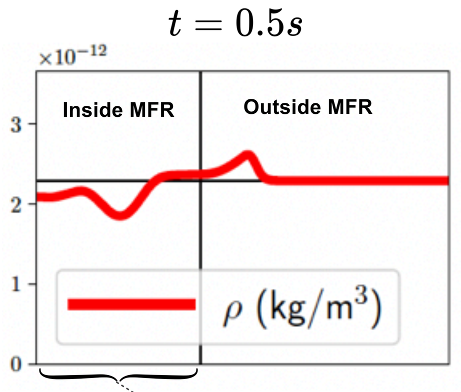
Rotational Magnetic Field
- $B_\theta = \left(\frac{r}{\bar{r}}\right)\frac{B_0}{1 + (r/\bar{r})^2}$

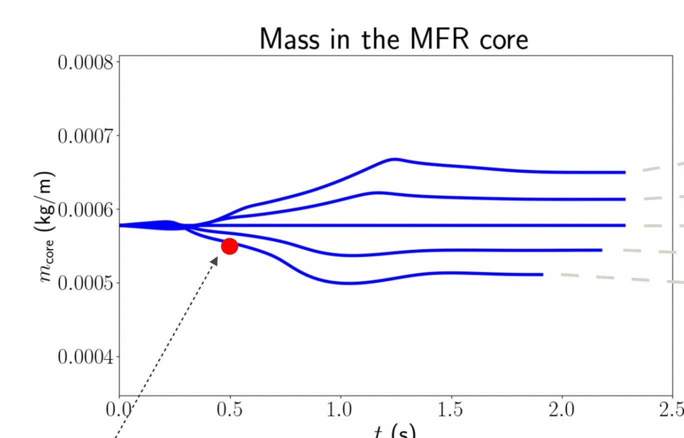


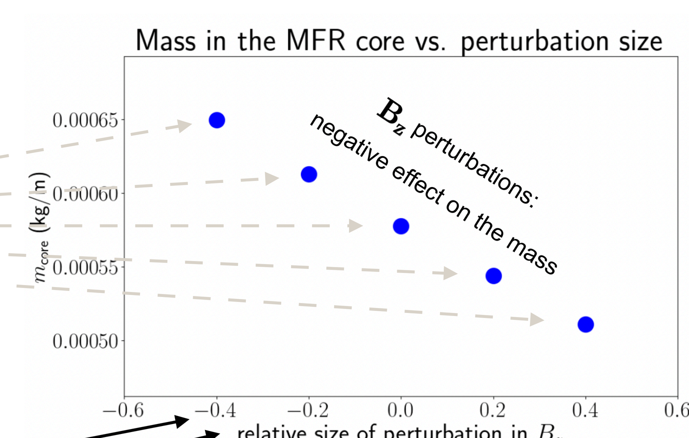
Dynamic Simulation

$t = 0\,(s)$ $t = 0.5s$ $t = 1.6s$







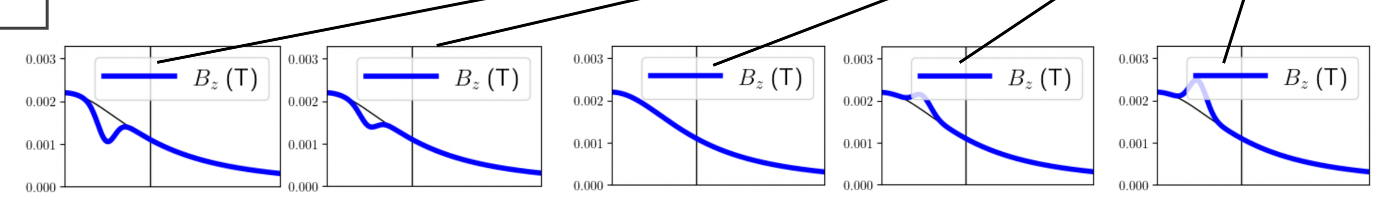


The x-axis is the size of the perturbation and the y-axis is the total mass.

The steep slope indicates that perturbations in $\mathbf{B_z}$ have a big effect on the mass of the MFR.

Integrate density to get total mass inside the MFR

The vertical bar indicates the edge of the MFR. I only integrated the density within the MFR, from the left side to the bar.



Negative perturbation ⟵ ⟶ Positive Perturbation

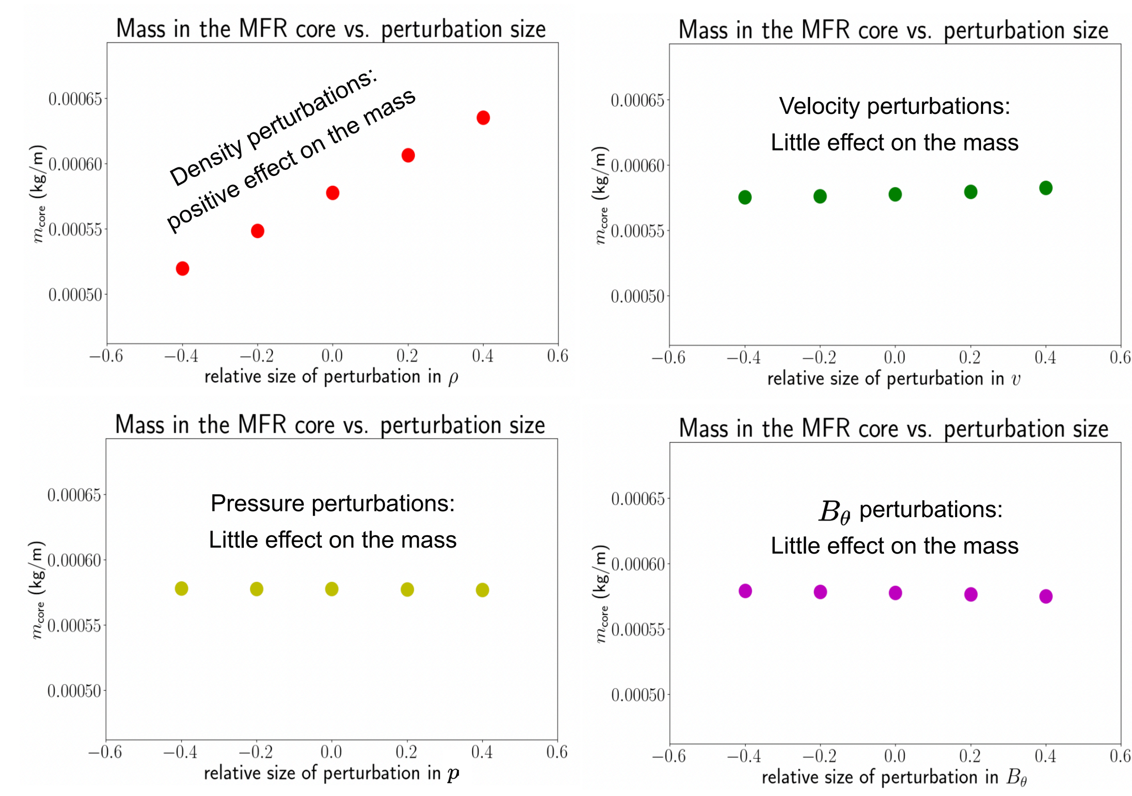
- **Computational Model**
  - Wrote a Python program to solve these five equations.
  - Used the numpy library.
  - Used the Lax-Wendroff finite difference scheme (second order).
  - Used vector to describe the different state variables.

$$u = [\rho,\, v_r,\, p,\, B_z,\, B_\theta]$$

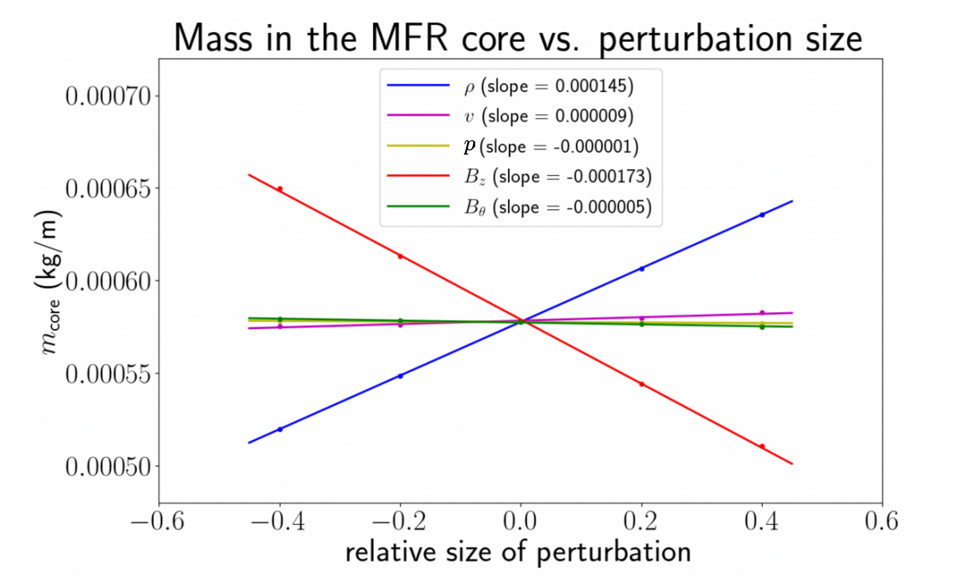
For $\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = f$, the Lax-Wendroff scheme is

$$u_j^{n+1} = u_j^n - a\left(\frac{\Delta t}{\Delta x}\right)\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{a^2}{2}\left(\frac{\Delta t}{\Delta x}\right)^2\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right) + \frac{\Delta t}{2}\left(f_j^{n+1} + f_j^n\right) - \frac{a\Delta t}{4}\left(\frac{\Delta t}{\Delta x}\right)\left(f_{j+1}^n - f_{j-1}^n\right)$$

(Results continued)



Conclusions



1. **Perturbations in $\mathbf{B_z}$ and ρ have the greatest effect on MFRs** while perturbations in B_θ, v_r, and p have little effect on MFRs. You can see this in the graph. B_z and ρ have big slopes while B_θ, v_r, and p have very small slopes.
2. The effects of the perturbation are also dependent on the magnitude and sign of the perturbation. If the MFR has a big, negative perturbation in B_z, you will have a big increase in mass. If the MFR has a small, positive perturbation in B_z, you will have small decrease in mass.
3. While B_z and ρ perturbations have bigger effect on MFRs, they don't change the MFR's general structure. This means MFRs are stable, but they do experience a change in mass for each perturbation.

On the surface of the sun, solar flares are constantly perturbed by outside forces, such as other magnetic fields and mass from other solar flares. According to my simulation, this means that **a solar flare can grow enormously or shrink to a minuscule size** based on outside forces. These findings align with real world observations of solar flares and ongoing attempts to better model and understand solar flares [1]. One thing to note is that my simulations were done in one dimension, r. If performed in 2D or 3D, then the results might be different.

Bibliography

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[5] Skinner, M., & Ostriker E., (2010, May 4) *The Athena Astrophysical Magnetohydrodynamics Code in Cylindrical Geometry.* The Astrophysical Journal Supplement Series.